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SOME COMMON FIXED POINT THEOREMS OF COMPATIBLE MAPS WITH INTEGRAL TYPE CONTRACTION IN G-METRIC SPACES

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Abstract. In this paper, we prove some common fixed point theorems in G-metric space by using the notion of integral type contraction. An example is also provided to illustrate our results.

Keywords: G-metric space, common fixed point, compatible maps, integral type contractive mapping.

AMS Subject Classification: 47H09, 47H10.

1. Introduction

In 2003, Z. Mustafa and B. Sims [16] introduced a more appropriate and robust notion of a generalized metric space. In such kind of spaces a nonnegative real number is assigned to every triplet of elements. In [17] they proved some fixed point results for mapping satisfying sufficient conditions on complete G-metric space. After that several other fixed point theorems have been proved in G-metric spaces by many researchers, see [2, 3, 4, 6, 7, 8, 9, 16, 24, 25, 27]. The studies relevant to metric spaces are being extended to G-metric spaces by several other researchers. For instance, we noted that a best approximation result in G-metric spaces established by Nezhad and Mazaheri in [19], the notion of w-distance, which is relevant to minimization problem in metric spaces [13], has been extended by Saadati et al. [22] to G-metric spaces. Also, Shatanawi [26] gave the concept of ordered generalized metric spaces and presented some fixed point results in ordered G-metric spaces. There has been an important interest to study common fixed point for a pair of mappings that satisfying some contractive conditions in metric spaces. Some elegant and interesting results were obtained in this direction by various authors. In 1976, G. Jungck [10] introduced the notion of commutativity and presented some common fixed point theorems. Also G. Jungck [11] introduced the concept of compatible mappings and proved fixed point results. It is noticed that the problems of fixed point of non-compatible mappings are very important and considered in a number of research studies, see [12, 21]. Also weaker version of commutativity has been considered in a large number of works. One such notion is R-weakly commutativity. This is an extension of weakly commuting mappings [20, 23].

2. Preliminaries

The following definitions and results will be needed in this paper.

Definition 1. [16] Let Y be a nonempty set, and let G: Y × Y × Y → R+ be a function satisfying the following axioms:

(G1) G (a, b, c) = 0 if a = b = c,

(G2) 0 < G (a, a, b), for all a, b ∈ Y with $\mathbf{a} \neq \mathbf{b}$,

(G3) G (a, a, b) ≤ G (a, b, c), for all a, b, c ∈ Y with $\mathbf{c} \neq \mathbf{b}$,

(G4) G (a, b, c) = G (a, c, b) = G(b, c, a) = . . . (symmetry in all variables),

(G5) G (a, b, c) ≤ G (a, s, s) + G(s, b, c), ∀ a, b, c, s ∈ Y, (rectangle inequality).

Then the function G is called a generalized metric, or more specifically a G-metric on Y, and the pair (Y, G) is called a G-metric space.

Example 1. [16] Let Y = {x, y}. Define G on Y × Y × Y by

G(x, x, x) = G(y, y, y) = 0, G(x, x, y) = 1, G(x, y, y) = 2

and extend G to Y × Y × Y by using the symmetry in the variables. Then it is clear that (Y, G) is a G-metric space.

Definition 2. [16] Let (Y, G) be a G-metric space and (an) a sequence of points of Y . A point a ∈ Y is said to be the limit of the sequence (a_n), if $\lim_{n,m\to\infty} G\left(a, a_n, a_m\right) = 0$ and we say that the sequence (a_n) is G-convergent to a.

Proposition 1. [16] Let (Y, G) be a G-metric space. Then the following are equivalent:

(1) (a_n) is G-convergent to a.

(2) G (a_n, a_n, a) → 0 as n → +∞.

(3) G (a_n, a, a) → 0 as n → +∞.

(2) G (a_n, a_m, a) → 0 as n, m → +∞.

Definition 3. [14] Let (Y, G) be a G-metric space. A sequence (an) is called G-Cauchy if for every $\in > \mathbf{0}$, there is k ∈ N such that $G\left(a_n,\ a_m,\ a_l\right) < \in$, for all n, m, l ≥ k; that is G(a_n, a_m, a_l) → 0 as n, m, l → +∞.

Proposition 2. [16] Let (Y, G) be a G-metric space. Then the following are equivalent:

(1) The sequence (an) is G-Cauchy.

(2) For every $\in > \mathbf{0}$, there is k ∈ N such that $G\left(a_n,\ a_m,\ a_l\right) < \in$, for all l, n, m ≥ k**.**

Definition 4. [16] A G-metric space (Y, G) is called G-complete if every G-Cauchy sequence in (Y, G) is G-convergent in (Y, G).

Proposition 3. [16] Let (Y, G) be a G-metric space. Then for any a, b, c, e ∈ Y, it follows that

(i) if G(a, b, c) = 0, then a = b = c;

(ii) G (a, b, c) ≤ G (a, a, b) + G (a, a, c);

(iii) G (a, b, b) ≤ 2G (b, a, a);

(iv) G (a, b, c) ≤ G (a, e, c) + G (e, b, c);

(v) G (a, b, c) ≤ 2 3(G (a, b, e) + G (a, e, c) + G (e, b, c));
(vi) G (a, b, c) ≤ G (a, e, e) + G (b, e, e) + G (c, e, e).

Proposition 4. [16] Let (Y, G) be a G-metric space. Then the function G (a, b, c) is jointly continuous in all three of its variables.

Proposition 5. [11] Let f and g be weakly compatible self-mappings on a set Y. If f and g have unique fixed point of coincidence w = fa = ga, then w is the unique common fixed point of f and g.

Definition 5. [11] Let f and g be two self-mappings on a metric space (Y, d). The mappings f and g are said to be compatible if $\lim_{n\to\infty} d\left(fga_n,\ gfa_n \right) = 0$, whenever $\{a_n\}$ is a sequence in Y such that $\lim_{n\to\infty} \mathbf{fa_n} = \lim_{n\to\infty} \mathbf{ga_n} = \mathbf{z}$ for some z ∈ Y.

Definition 6. [4] Let (Y, G) be a G-metric space and H: Y → Y be a self-mappings on (Y, G). Now H is said to be a contraction if

$$G (Ha, Hb, Hc) \le \alpha\, G (a, b, c) \text{ for all } a, b, c \in Y \text{ where } \alpha \in [0, 1). \tag{1}$$

Clearly every self-mapping H: Y → Y satisfying condition (1) is continuous. To generalize the condition (2.1) for a pair of self-mappings S and H on Y :

$$G (Sa, Sb, Sc) \le \alpha\, G (Ha, Hb, Hc) \text{ for all } a, b, c \in Y \text{ where } \alpha \in [0, 1). \tag{2}$$

Definition 7. [4] let f and g be two self-mappings on a G-metric space (Y, G). The two mappings are said to be compatible if $\lim_{n\to\infty} G\left(fga_n,\ gfa_n,\ gfa_n \right) = 0$, whenever $\{a_n\}$ is a sequence in Y such that $\lim_{n\to\infty} fa_n = \lim_{n\to\infty} ga_n = z$ for some z ∈ Y.

In 2002, Branciari in [5] introduced a general contractive condition of integral type as follows.

Theorem 1. [5] Let (Y, d) be a complete metric space, α ∈ (0, 1), and f: Y → Y is a mapping such that for all x, y ∈ Y ,

$$\int_0^{\mathbf{d}\left(\mathbf{f}(\mathbf{x}),\,\mathbf{f}(\mathbf{y})\right)} \varphi(t)dt \le \alpha \int_0^{\mathbf{d}(\mathbf{x},\mathbf{y})} \varphi(t)dt .$$

where φ: [0, +∞) → [0, +∞) is nonnegative and Lebesgue-integrable mapping which is summable (i.e., with finite integral) on each compact subset of [0, +∞) such that for each **ϵ** > 0, $\int_0^{\epsilon} \varphi(t)dt$, then f has a unique fixed point a ∈ Y , such that for each x ∈ Y , $\lim_{n\to\infty} \mathbf{f^n}\left(\mathbf{x}\right) = \mathbf{a}$.

The aim of this research paper is to carry the above idea of integral type contractive mappings to G-metric spaces.

3. Main results

In this section, we prove some common fixed point results in the setting of G-metric spaces by using the idea of integral type contractive mappings. Our first main result is stated as:

Theorem 2. Let (Y, G) be a complete G-metric space and f, g be two self-mappings on (Y, G) satisfies the following conditions:

(1) $f(Y) \subseteq g(Y)$, (3)

(2) f or g is continuous, (4)

(3)

$$\int_0^{G(fa,fb,fc)} \varphi(t)dt \le \boldsymbol{\alpha} \int_0^{G(fa,gb,gc)} \varphi(t)dt + \boldsymbol{\beta} \int_0^{G(ga,fb,gc)} \varphi(t)dt + \boldsymbol{\gamma} \int_0^{G(gc,gb,fc)} \varphi(t)dt. \quad (5)$$

For every a, b, c ∈ Y and $\alpha, \beta, \gamma \ge 0$ with $0 \le \alpha + 3\beta + 3\gamma < 1$ and $\varphi: [0,+\infty) \to [0,+\infty)$ is a Lebesgue integrable mapping which is summable, non-negative and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \varphi(t)dt > 0$. Then the mappings f and g have a unique common fixed point in Y provided f and g are compatible maps**.**

Proof. Let a_0 be arbitrary in Y. Choose $a_1 \in Y$ such that $fa_0 = ga_1$. In general we can choose a_{n+1} such that $b_n = fa_n = ga_{n+1}$, n = 0,1,2, ...
From (5), we have

$$\int_0^{G(fa_n,fa_{n+1},fa_{n+1})} \varphi(t)dt \le \alpha \int_0^{G(fa_n,ga_{n+1},ga_{n+1})} \varphi(t)dt + \beta \int_0^{G(ga_n,fa_{n+1},ga_{n+1})} \varphi(t)dt + \gamma \int_0^{G(ga_n,ga_{n+1},fa_{n+1})} \varphi(t)dt$$

$$= \alpha \int_0^{G(fa_n,fa_n,fa_n)} \varphi(t)dt + \beta \int_0^{G(fa_{n-1},fa_{n+1},fa_n)} \varphi(t)dt + \gamma \int_0^{G(fa_{n-1},fa_n,fa_{n+1})} \varphi(t)dt.$$

$$= (\beta + \gamma) \int_0^{G(fa_{n-1},fa_n,fa_{n+1})} \varphi(t)dt.$$

By use of (G5) and Proposition 3, we have

$G(fa_{n-1}, fa_n, fa_{n+1}) \leq G(fa_{n-1}, fa_n, fa_n) + G(fa_n, fa_n, fa_{n+1})$

$\leq G(fa_{n-1}, fa_n, fa_n) + 2G(fa_n, fa_{n+1}, fa_{n+1})$.

Then,

$$\int_0^{G(fa_n, fa_{n+1}, fa_{n+1})} \varphi(t)\,dt \leq (\beta+\gamma) \int_0^{G(fa_{n-1}, fa_n, fa_{n+1})} \varphi(t)\,dt$$

$$\leq \Big\{ (\beta+\gamma) \int_0^{G(fa_{n-1}, fa_n, fa_n)} \varphi(t)\,dt + \int_0^{2G(fa_n, fa_{n+1}, fa_{n+1})} \varphi(t)\,dt \Big\}$$

$$\leq (\beta+\gamma) \int_0^{G(fa_{n-1}, fa_n, fa_n)} \varphi(t)\,dt + (2\beta+2\gamma) \int_0^{2G(fa_n, fa_{n+1}, fa_{n+1})} \varphi(t)\,dt$$

$$\leq \frac{(\beta+\gamma)}{(1-2\beta-2\gamma)} \int_0^{G(fa_{n-1}, fa_n, fa_n)} \varphi(t)\,dt$$

$$\leq l \int_0^{G(fa_{n-1}, fa_n, fa_n)} \varphi(t)\,dt.$$

where $l = \dfrac{(\beta+\gamma)}{(1-2\beta-2\gamma)} < 1$.

Continuing this process, we get

$$\int_0^{G(fa_n, fa_{n+1}, fa_{n+1})} \varphi(t)\,dt \leq l^n \int_0^{G(fa_0, fa_1, fa_1)} \varphi(t)\,dt$$

For all n, m ∈ N, n < m, we have

$$\int_0^{G(b_n, b_m, b_m)} \varphi(t)\,dt \leq \int_0^{G(b_n, b_{n+1}, b_{n+1})} \varphi(t)\,dt + \int_0^{G(b_{n+1}, b_{n+2}, b_{n+2})} \varphi(t)\,dt$$

$$+\cdots+\int_0^{G(b_{m-1},b_m,b_m)} \varphi(t)\,dt\,.$$

$$\leq (l^n+l^{n+1}+....+l^{m-1}) \int_0^{G(b_0,b_1,b1)} \varphi(t)\,dt$$

$$\leq \frac{l^n}{(\mathbf{1}-l)} \int_0^{G(b_0,b_1,b1)} \varphi(t)\,dt \to 0 \text{ as n, m} \to \infty.$$

Thus,

$$\lim_{n,m\to\infty} G(b_n,\, b_m,\, b_m)=0\,.$$

This means that $\{b_n\}$ is a G-Cauchy sequence in Y. Since (Y, G) is complete G-metric space, therefore, there exists a point p ∈ Y such that $\lim_{n\to\infty} b_n=\lim_{n\to\infty} fa_n=\lim_{n\to\infty} ga_{n+1}=p.$

As the mapping f or g is continuous, so we can assume that g is continuous, therefore $\lim_{n\to\infty} gfa_n=\lim_{n\to\infty} gga_n=gp$.

Also f and g are compatible, therefore, $\lim_{n\to\infty} G(\,fga_n,\; gfa_n,\; gfa_n)=0$, this implies $\lim_{n\to\infty} f\mathbf{ga_n}=\mathbf{g}p$.

From (5), we have

$$\int_0^{G(fga_n,fa_n,fa_n)} \varphi(t)\,dt \leq \alpha \int_0^{G(fga_n,ga_n,ga_n)} \varphi(t)\,dt + \beta \int_0^{G(gga_n,fa_n,ga_n)} \varphi(t)\,dt$$

$$+\gamma \int_0^{G(gga_n,ga_n,fa_n)} \varphi(t)\,dt\;.$$

Taking limit as n → ∞, we have gp = p.

Again from condition (5), we have

$$\int_0^{G(fa_n,f_p,f_p)} \varphi(t)\,dt \leq \boldsymbol{\alpha} \int_0^{G(fa_n,g_p,g_p)} \varphi(t)\,dt + \boldsymbol{\beta} \int_0^{G(ga_n,f_p,g_p)} \varphi(t)\,dt$$

$$+\boldsymbol{\gamma} \int_0^{G(ga_n,g_p,f_p)} \varphi(t)\,dt\,\mathbf{.}$$

By taking limit as n → ∞, we have p = fp. Therefore, we have gp = fp = p. Thus p is a common fixed point of f and g.

For uniqueness, we suppose that p1 6= p be another common fixed point of f and g Then

$$\int_0^{G(p,p_1,p_1)} \varphi(t)\,dt = \int_0^{G(fp,fp_1,fp_1)} \varphi(t)\,dt$$

$$\leq \alpha \int_0^{G(fp,gp_1,gp_1)} \varphi(t)\,dt + \beta \int_0^{G(gp,fp_1,gp_1)} \varphi(t)\,dt$$

$$+ \gamma \int_0^{G(gp,gp_1,fp_1)} \varphi(t)\,dt$$

$$= (\alpha + \beta + \gamma) \int_0^{G(p,p_1,p_1)} \varphi(t)\,dt$$

$$< \int_0^{G(p,p_1,p_1)} \varphi(t)\,dt .$$

This arise contradiction and hence $p_1 = p$. The proof is completed.

Corollary 1. Let (Y, G) be a complete G-metric space and f, g be two compatible self-mappings on (Y, G) satisfies assertions (3), (4) and the following condition**:**

$$\int_0^{G(fa,fb,fc)} \varphi(t)\,dt \leq l \int_0^{G(a,b,c)} \varphi(t)\,dt ,$$

for all a, b, c ∈ Y and $0 < l < 1$. Then f and g have a unique common fixed point in Y.

Theorem 3. Let f and g be two weakly compatible self-mappings of a Gmetric space (Y, G) satisfying conditions (3) and (5) and any one of the subspace f(Y) or g(Y) is complete. Then f and g have a unique common fixed point in Y **.**

Proof. From the main result 3, we conclude that $\{b_n\}$ is a G-Cauchy sequence in Y. Since either f(Y) or g(Y) is complete, we assume that g(Y) is complete subspace of Y then the subsequence of $\{b_n\}$ must get a limit in g(Y) be p. Let $v \in g^{-1}p$. Then gv = p as $\{b_n\}$ is a G-Cauchy sequence containing a convergent subsequence, therefore the sequence $\{b_n\}$ also convergent implying thereby the convergence of subsequence of the convergent sequence. Now we can show that fv = p.

Setting a = v, b = a_n and p = a_n, in condition (5), we have

$$\int_0^{G(fv,fa_n,fa_n)} \varphi(t)\,dt \leq \alpha \int_0^{G(fv,ga_n,ga_n)} \varphi(t)\,dt + \beta \int_0^{G(gv,fa_n,ga_n)} \varphi(t)\,dt$$

$$+ \gamma \int_0^{G(gv,ga_n,fa_n)} \varphi(t)\,dt .$$

As n → ∞ in above inequality, we get

$$\int_0^{G(fv,p,p)} \varphi(t)\,dt \le \alpha \int_0^{G(fv,p,p)} \varphi(t)\,dt .$$

Implies that fv = p.

Therefore, fv = gv = p, that is, v is a coincident point of two mappings f and g. Since the two mappings f and g are weakly compatible, it follows that fgv = gfv, that is, fp = gp.

Next we show that fp = p. Further we assume that $\mathbf{fp} \ne \mathbf{p}$.

From condition (5), we set a = p, b = v, p = v, we have

$$\int_0^{G(fp,p,p)} \varphi(t)\,dt = \int_0^{G(fp,fv,fv)} \varphi(t)\,dt$$

$$\le \alpha \int_0^{G(fp,gv,gv)} \varphi(t)\,dt + \beta \int_0^{G(gp,fv,gv)} \varphi(t)\,dt$$

$$+ \gamma \int_0^{G(gp,gv,fv)} \varphi(t)\,dt .$$

$$= (\alpha + \beta + \gamma) \int_0^{G(fp,p,p)} \varphi(t)\,dt$$

$$< \int_0^{G(fp,p,p)} \varphi(t)\,dt .$$

Which is contradiction and hence fp = p. Therefore, fp = gp = p that is, p is common fixed point of mappings f and g. We can show the uniqueness as above easily. The proof is completed.

We now give an example to illustrate Theorem 2.

Example 2. Suppose that Y = [0,1] and also assume that G be the G-metric on Y ×Y ×Y defined as G(a, b, c) = |a−b|+|b−c|+|c−a| ∀ a, b, c ∈ Y .

Then (Y, G) be a G-metric space. We define $\mathbf{fa} = \frac{\mathbf{a}}{\mathbf{6}}$ and $\mathbf{ga} = \frac{\mathbf{a}}{2}$. Also we noted that, the mapping f is continuous and f(Y) ⊆ g(Y). Also,

$$\int_0^{G(fa,fb,fc)} \varphi(t)\,dt \le l \int_0^{G(ga,gb,cc)} \varphi(t)\,dt ,$$

holds for all a, b, c ∈ Y , $\frac{\mathbf{1}}{\mathbf{3}} \le l < 1$ and 0 is the unique common fixed point of f and g.

References

1. Aamri M, Moutawakil D.E., Some new common fixed point theorems under strict contractive conditions, J. Math. Anal. Appl., Vol.270, 2002, pp.181–188.
2. Abbas M, Rhoades B. E., Common fixed point results for noncommuting mapping without continuity in generalized metric spaces, Appl. Math. Comput., Vol.215, 2009, pp.262–269.
3. Aydi H., Shatanawi W., Vetro C., On generalized weakly G-contraction mapping in G-metric Spaces, Comput. Math. Appl., Vol.62, 2011, pp.4222–4229.
4. Binayak S., Choudhury, Sanjay K., Asha, Krishnapada D., Some fixed point theorems in G-metric Spaces, Math. Sci. lett., Vol.1, 2012, pp.25–31.
5. Branciari A., A fixed point theorem for mappings satisfying a general contractive condition of integral type, Int. J. of Math. and Math. Sci.,Vol.29, No.9, 2002, pp.531–536.
6. Choudhury B. S., Maity P., Coupled fixed point results in generalized metric spaces, Math. Comput. Modelling, Vol.54, 2011, pp.73–79.
7. Gupta V., Deep R., Common fixed point theorem involving two mappings and weakly contractive condition in G-metric spaces, Proceedings of the Institute of Applied Mathematics, Vol.3, No.1, 2014, pp.80–88.
8. Gupta V., Mani N., Existence and uniqueness of fixed point for contractive mapping of integral type, International Journal of Computing Science and Mathematics, Vol.4, No.1, 2013, pp.72–83.
9. Gupta V., Mani N., Tripathi A. K., A fixed point theorem satisfying a generalized weak contractive condition of integral type, International Journal of Mathematical Analysis, Vol.6, No.38, 2012, pp.1883–1889.
10. Jungck G., Commuting mappings and fixed points, Amer. Math.Monthly., Vol.83, 1976, pp.261–263.
11. Jungck G., Compatible mappings and common fixed points, Int. J. Math. and Math. Sci., Vol. 9, No.4, 1986, pp.771–779.
12. Jungck G., Hussain N., Compatible maps and invariant approximations, J. Math. Anal. Appl., Vol. 325, No.2, 2007, pp.1003–1012.
13. Kada O., Suzuki T., Takahashi W., Nonconvex minimization theorems and fixed point theorems in complete metric spaces, Math. Japon., Vol.44, 1996, pp.381–391.
14. Murthy P.P., Tas K., New common fixed point theorems of Gregus type for R- weakly commuting mappings in 2-metric spaces, Hacet. J. Math. Stat., Vol.38, 2009, pp.285–291.
15. Mustafa Z., Sims B., Some remarks concerning D-metric spaces, Proceedings of International Conference on Fixed Point Theory and Applications, pp. 189–198, Yokohama, Yokohama, Japan, 2004.
16. Mustafa Z., Sims B., A new approach to generalized metric spaces, J.

Nonlinear Convex Anal., Vol.7, No,2, 2006, pp.289–297.
17. Mustafa Z., Obiedat H., Awawdeh F., Some fixed point theorem for mapping on complete G-metric spaces, Fixed Point Theory and Applications,Vol.2008,ArticleID 189870, doi:10.1155/2008/189870.
18. Mustafa Z., Sims B., Fixed point theorems for contractive mappings in complete G-metric spaces, Fixed Point Theory and Applications, Vol. 2009, Article ID 917175, doi:10.1155/2009/917175.
19. Nezhad A. D., Mazaheri H., New results in G-best approximation in G-metric spaces, Ukrainian Math. J., Vol.62, 2010, pp.648–654.
20. Pant R. P., Common fixed points of weakly commuting mappings, Math. Student, Vol.62, 1993, pp.97–102.
21. Pant R.P., Common fixed points of contractive maps, J. Math. Anal. Appl., Vol.226, 1998, pp.251–258.
22. Saadati R., Vaezpour S. M., Vetro P., Rhoades B.E., Fixed point theorems in generalized partially ordered G-metric spaces, Math. Comput. Modelling, Vol.52, 2010, pp.797–801.
23. Sessa S., On a weak commutativity conditions of mappings in fixed point considerations, Publ. Inst. Math. Beograd, Vol.32, No.46, 1982, pp.146–153.
24. Shah R., Zada A., Khan I., Some fixed point theorems of integral type contraction in cone b-metric spaces, Turkish. J. Ana. Num. Theor., Vol.3, No.6, 2105, pp.165–169.
25. Shah R., Zada A., Li T., New common coupled fixed point results of integral type contraction in generalized metric spaces, J. Ana. Num.Theor., Vol.4, No.2, 2106, pp.1–8.
26. Shatanawi W., Some fixed point theorems in ordered G-metric spaces and applications, Abst. Appl. Anal., Vol.2011, 2011, Article ID:126205.
27. Zada A., Shah R., Li T., Integral type contraction and coupled coincidence fixed point theorems for two pairs in G-metric spaces, Hacet. J. Math. Stat., Doi: 10.15672/HJMS.20164514280.

G-metrik fəzalarda inteqral tipli sıxılmalarla uzlaşmalı inikaslarda tərpənməz nöqtə haqqında bəzi ümumi teoremlər

Rəhim Şah, Əkbər Zadə

XÜLASƏ

Məqalədə G-metrik fəzalarda inteqral tip sıxılmalardan istifadə etməklə tərpənməz nöqtələr haqqında bəzi ümumi teoremlər isbat edilir. Alkınan nəticələri illüstrasiya edən misallar verilmişdir.

Açar sözlər: G-metrik fəzalar, ümumi tərpənməz nöqtə, uzlaşan inikaslar, inteqral tip sıxılmış inikas.

Некоторые общие теоремы о неподвижных точках совместимых отображений с интегрального типа сжатиями в G-метрических пространствах

Рахим Шах, Акбар Zada

РЕЗЮМЕ

В данной работе мы докажем некоторые общие теоремы о неподвижной точке в G-метрическом пространстве, используя понятие сжатия интегрального типа. Дается пример для иллюстрации наших результатов.

Ключевые слова: G-метрические пространства, общие неподвижные точки, совместимые отображения, отображения с интегрального типа сжатиями