COERCIVE ESTIMATION OF THE SOLUTIONS OF INFINITE SYSTEM OF INTEGRO-DIFFERENTIAL EQUATIONS IN WEIGHTED SPACES*

H.K. Musaev

Baku State University, Department of Differential and Integral Equation
Baku, Azerbaijan
e-mail: gkm55@mail.ru

Abstract. The some properties of convolution differential-operator equations in a Banach space are studied. Also it is established that the corresponding realization operator is positive and R-positive. These results permit us to show the separability of the differential operators in a E-valued weighted Lp-space. In the present paper, by using the positivity and separability properties of the convolution-elliptic operators of the infinite systems of integro-differential equations are obtained.

Keywords: Infinite system, integro-differential equations, R-positivity, weighted multiplier condition, convolution equations.

AMS Subject Classification:

1. Introduction

Boundary value problems (BVPs) for differential-operator equations (DOE) have been studied in [1-3], [7], [10], [18]. Operator-valued Fouirer multipliers have been investigated in [4-9], [17]]. In the present paper we consider the E −valued weighted spaces $L_{p,\gamma}(\mathbb{R};E)$, here $\gamma=\gamma(x)$ is a weighted function from $A_p,\ p\in(1,\infty)$.

In recent years separability properties of elliptic convolution operators in weighted spaces have been studied extensively e.g. in [1], [12], [15]. Convolution-differential equations (CDEs) have been treated e.g. in [11], [13]. Moreover, the convolution differential-operator equations (CDOEs) is relatively less investigated subject. In [2], [7] and [14] the parabolic type CDEs with operator coefficient were investigated L_p −spaces. In the paper the main aim to study the following infinite system of convolution equation

$$\sum_{k=0}^{l} a_k * \frac{d^k u_i}{dx^k} + \sum_{j=1}^{\infty}\left(b_{ij}+\lambda\right) * u_j(x) = f_i(x) \qquad (1)$$

in $L_{p,\gamma}(\mathbb{R};l_q)$where $x\in\mathbb{R}, i=1,2,\dots, a_k=a_k(x)$ are complex numbers, $q\in(1,\infty)$.

Further, we will consider the following infinite system of Cauchy problem for

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parabolic CDEs

$$\frac{\partial u_j}{\partial t}+\sum_{k=0}^{l} a_k * \frac{\partial^k u_i}{\partial x^k}+\sum_{j=1}^{\infty} b_{ij} * u_j(t,x) = f_i(t,x), \quad (2)$$

$$u_j(0,x)=0, x\in\mathbb{R}.$$

We say that the problem (1) is $L_{p,\gamma}(\mathbb{R};l_q)$ −separable, if for all $f=\{f_i\}\in L_{p,\gamma}(\mathbb{R};l_q)$ there exists a unique solution $u=\{u_i\}\in W_{p,\gamma}^{l}(\mathbb{R};l_q(B),l_q)$ of the problem (1) satisfying this problem almost everywhere on $\mathbb{R}$ and there exists a positive constant C independent on f, such that one has the coercive estimate

$$\sum_{k=0}^{l}|\lambda|^{1-\frac{k}{l}}\left\|a_k * \frac{d^k u_i}{dx^k}\right\|_{L_{p,\gamma}(\mathbb{R};l_q)}+\|B*u\|_{L_{p,\gamma}(\mathbb{R};l_q)} \le C\|f\|_{L_{p,\gamma}(\mathbb{R};l_q)}. \quad (3)$$

The (3) implies that if $f\in L_{p,\gamma}(\mathbb{R};l_q)$ and u is the solution of the BVPs (1) then all terms of (1) are separable in $L_{p,\gamma}(\mathbb{R};l_q)$.It means that the inverse of the realization operator generated (1.1) is bounded from $L_{p,\gamma}(\mathbb{R};l_q)$to $W_{p,\gamma}^{l}(\mathbb{R};l_q(B),l_q)$.

The paper is organized as follows. In section 2 we collect the necessary tools from weighted Banach space theory, weighted multiplier condition, R −positivity and some background material is given. In section 3 the separability properties for (1) are established and by applying this result the coercive estimate of infinite systems of integro-differential equations is proved.

2. Notations and background

Let $\mathbb{N},\mathbb{R}$ and $\mathbb{C}$ denote the sets of natural, real and complex numbers, respectively. If E_1 and E_2 are Banach spaces, $\mathcal{L}(E_1,E_2)$ denotes the Banach space of all bounded linear operators from E_1 into E_2 with the norm equal to the operator norm. For $E_1=E_2=E$, we write $\mathcal{L}(E)=\mathcal{L}(E,E)$.

$\mathbb{R}^n$denotes the n −dimensional Euclidean space , dx is the corresponding Lebesgue measure. For 1<q<∞ we set

$$l_q=\left\{\xi;\xi=\{\xi_i\}_{i=1}^{\infty},\|\xi\|_{l_q}=\left(\sum_{i=1}^{\infty}|\xi_i|^q\right)^{1/q},\xi_i-complex\ numbers\right\}.$$

Let $\gamma=\gamma(x), x=(x_1,x_2,\dots,x_n)$ be a positive measurable real-valued function on a measurable subset $\Omega\subset\mathbb{R}^n$. By the sumbol $L_{p,\gamma}(\Omega;\mathrm{E})$ we mean the space of all strongly E −valued functions on a $\Omega\subset\mathbb{R}^n$with norm

$$\|f\|_{L_{p,\gamma}} = \|f\|_{L_{p,\gamma}(\Omega;\mathrm{E})} = \left(\int_\Omega \|f(x)\|_E^p \gamma(x)dx\right)^{1/p}, 1 \le \mathrm{p} < \infty.$$

For $\gamma(x) \equiv 1$, the space $L_{p,\gamma}(\Omega; \mathrm{E})$ will be denoted by $L_p = L_{p,\gamma}(\Omega; \mathrm{E})$.

$$\|f\|_{L_{\infty,\gamma}(\Omega;\mathrm{E})} = \operatorname*{esssup}_{\mathrm{x}\in\Omega}[\gamma(\mathrm{x})\|f(x)\|_E].$$

Moreover, if $\gamma(x)$ is a positive measurable function, then for $1 < p < \infty$,

$$L_{p,\gamma}(l_q) = \left\{f; f = \{f_i(x)\}_{i=1}^\infty, \|f\|_{L_{p,\gamma}(l_q)} = \left(\int_{\mathbb{R}^n} \gamma(x)\|f_i(x)\|_{l_q}^p dx\right)^{1/p} < \infty\right\}.$$

Clearly, $L_{p,\gamma}(l_q)$is a Banach space.

The weight function $\gamma(x)$ is said to satisfy an A_p condition, i.e., $\gamma(x) \in A_p, 1 < p < \infty$ if there is a positive constant C such that

$$\left(\frac{1}{|Q|}\int_Q \gamma(x)dx\right) = \left(\frac{1}{|Q|}\int_Q \gamma^{-\frac{1}{p-1}}(x)dx\right)^{p-1} \le C$$

for all compact sets $Q \subset \mathbb{R}^n$.

Example: By virtue of [16] the following weighted functions

$$\gamma(x) = |x|^\alpha, x \in \mathbb{R}, -1 < \alpha < p-1,$$

$$\gamma(x) = \prod_{k=1}^N \left(1 + \sum_{j=1}^n |x_j|^{\alpha_{jk}}\right)^{\beta_k}, 1 < p < \infty,$$

belong to A_p class, where α_{jk}, $N \in \mathbb{N}$, $\beta_k \in \mathbb{R}$.

Suppose that

$$S_\varphi = \{\lambda;\ \lambda \in \mathbb{C}, |arg\lambda| \le \varphi\} \cup \{0\}, 0 \le \varphi < \pi.$$

A closed linear operator function $A = A(x), x \in \mathbb{R}$ is said to be uniformly φ −positive in Banach space E, if $D(A(x))$ is dense in E and does not depend on x and there is a positive constant M so that

$$\|(A(x) + \lambda I)^{-1}\|_{\mathcal{L}(E)} \le M(1 + |\lambda|)^{-1},$$

for every $x \in \mathbb{R}$ and $\lambda \in S_\varphi, \varphi \in [0, \pi)$, where I is an identity operator in E. For a scalar λ, we sometimes write $A + \lambda$ or A_λ instead of $A + \lambda I$.

By $S = S(\mathbb{R}^n; E)$ we denote the Schwartz space of rapidly decreasing smooth E −valued functions on $\mathbb{R}^n$. $S'(\mathbb{R}^n; E)$we denote the space of linear continuous mappings from S to E which is called the Schwartz space of E −valued distributions. Recall $S(\mathbb{R}^n; E)$ is norm dense in $L_{p,\gamma}(\mathbb{R}^n; \mathrm{E})$.when$1 \le p < \infty, \gamma \in$

A_p

Let Ω be a domain in $\mathbb{R}^n$. $C(\Omega, E)$ and $C^{(m)}(\Omega; E)$will denote the spaces of E −valued bounded, uniformly strongly continuous and m −times continuously differentiable functions on Ω, respectively. For $E = \mathbb{C}$ the space $C^{(m)}(\Omega, E)$ will be denoted by $C^{(m)}(\Omega)$.

An E −valued generalized function D^α is called a generalized derivative in the sense of Schwartz distributions of the function $f \in S'(\mathbb{R}^n, E)$, if the equality

$$\langle D^\alpha f, \varphi \rangle = (-1)^{|\alpha|} \langle f, D^\alpha \varphi \rangle$$

holds for all $\varphi \in S$, where,

$$\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n), |\alpha| = \sum_{k=1}^{n} \alpha_k, D^\alpha = D_1^{\alpha_1}, D_2^{\alpha_2}, \dots, D_n^{\alpha_n}$$

α_i are integers.

Let F denote the Fourier transform. Through this section the Fourier transformation of a function f will be denoted by $\hat{f}, Ff = \hat{f}$ and $F^{-1}f = \check{f}$.

A function $\Psi \in L_\infty(\mathbb{R}^n; \mathcal{L}(E_1, E_2))$ is called a multiplier from $L_{p,\gamma}(\mathbb{R}^n; E_1)$ to $L_{p,\gamma}(\mathbb{R}^n; E_2)$for $p \in (1, \infty)$ if the map $u \to Bu = F^{-1}\Psi(\xi)Fu, u \in S(\mathbb{R}^n; E_1)$ are well defined and extends to a bounded linear operator

$$B: L_{p,\gamma}(\mathbb{R}^n; E_1) \to L_{p,\gamma}(\mathbb{R}^n; E_2)$$

The collection of all Fourier multipliers from $L_{p,\gamma}(\mathbb{R}^n; E_1)$to $L_{p,\gamma}(\mathbb{R}^n; E_2)$ will be denoted by $M_{p,\gamma}^{p,\gamma}(E_1, E_2)$. For $E_1 = E_2 = E$ it is simply denoted by $M_{p,\gamma}^{p,\gamma}(E)$. Let $M(h)$ denote a set of some parameters.

Consider the family $B_h = \{\Psi_h;\ \Psi_h \in M_{p,\gamma}^{p,\gamma}(E_1, E_2), h \in M(h)\}$ of multipliers from the collection $M_{p,\gamma}^{p,\gamma}(E_1, E_2)$. The multipliers Ψ_h are said to be uniformly bounded (UBM) with respect to h if there exists a positive constant M independent of $h \in M(h)$such that

$$\|F^{-1}\Psi_h F u\|_{L_{p,\gamma}(\mathrm{R}^n; E_2)} \le M \|u\|_{L_{p,\gamma}(\mathrm{R}^n; E_1)},$$

for all $h \in M(h)$ and $u \in S(\mathbb{R}^n; E_1)$.

The Banach space E is called UMD -space ([6],[17]) if the Hilbert operator of a function $f \in S(\mathbb{R}; E)$, is defined by $Hf = \frac{1}{\pi} PV\left(\frac{1}{t}\right) * f$, i.e.,

$$(Hf)(t) = \frac{1}{\pi} \lim_{\varepsilon \to 0} \int_{|\tau| > \varepsilon} \frac{f(t-\tau)}{\tau} d\tau$$

is bounded in $L_p(\mathbb{R}; E)$, for $p \in (1, \infty)$ (see e.g. [5], [9]). UMD spaces include e.g. L_p, l_pspaces, Hilbert spaces, Sobolev spaces and Lorentz spaces $L_{p,q},\ p, q \in (1, \infty)$.

A family of operators $T \subset \mathcal{L}(E_1, E_2)$ is called R −bounded (see [5], [17]) if there is a constant $C > 0$ such that for all $T_1, T_2, \dots, T_n \in T$ and $u_1, u_2, \dots, u_n \in E_1, n \in \mathbb{N}$

$$\int_0^1 \left\| \sum_{j=1}^n r_j(y) T_j u_j \right\|_{E_2} dy \le C \int_0^1 \left\| \sum_{j=1}^n r_j(y) u_j \right\|_{E_1} dy,$$

where$\{r_j\}$is a sequence of independent symmetric $\{-1;1\}$ −valued random variables on [0,1]. The smallest C for which the above estimate holds is called an R −bound of the collection T and denoted by $R(T)$.

Note that from the definition of R −boundedness it follows that every R −bounded family of operators is (uniformly) bounded (it is enough to take $n = 1$).

A Banach space E is said to be a space satisfying a weighted multiplier condition with respect to $p \in (1,\infty)$ and weighted function γ if, for any $\Psi \in L_\infty(\mathbb{R}, \mathcal{L}(E))$ the R −boundedness of the set

$$\{|\xi|^k D^k \Psi(\xi): \xi \in \mathbb{R}\backslash\{0\}, \qquad k = 0,1\}$$

implies that Ψ is a Fourier multiplier in $L_{p,\gamma}(\mathbb{R}^n; E)$, i.e. $\Psi \in M_{p,\gamma}^{p,\gamma}(E)$ for any $p \in (1,\infty)$. If $E = \mathbb{C}$ and $\gamma \in A_p, p \in (1,\infty)$ then $\Psi \in M_{p,\gamma}^{p,\gamma}(\mathbb{C})$.

Note that, if E is UMD space and $\gamma(x)$ then by virue of [5], [9] and [17] the space E satisfies the multiplier condition. The UMD spaces satisfy the uniformly multiplier condition.

It is well known (see [5]) that any Hilbert space satisfies the multiplier condition. There are, however, Banach spaces which are not Hilbert spaces but satisfy the multiplier condition.

A positive operator A is said to be R −positive in the Banach space E if there exists $\varphi \in [0,\pi)$ such that the set

$$\{\xi(A + \xi I)^{-1}; \xi \in S_\varphi\}$$

is R −bounded.

Note that, in a Hilbert space every bounded set is R −bounded. Therefore, in a Hilbert space, the notion of R −boundedness is a equivalent to boundedness of a family of operators and in a Hilbert spaces all positive operators are R −positive (see e.g. [5], [17]).

Let $A = A(x), x \in \mathbb{R}$, be closed linear operator in E with domain definition $D(A)$ independent of x and $u \in L_p(\mathbb{R}; E(A))$.Then define

$$(A * u)(x) = \int_{\mathbb{R}} A(x - y) u(y) dy.$$

We consider the E −valued weighted space

$$W_{p,\gamma}^l(\mathbb{R}^n; E_0, E) = \{u; u \in L_{p,\gamma}(\mathbb{R}^n; E_0),\ \ D_k^{l_k} u \in L_{p,\gamma}(\mathbb{R}^n; E)\},$$

where $l = (l_1, l_2, \dots, l_n), l_k$ −are positive integers, $D_k^{l_k} = \frac{\partial^{l_k}}{\partial_{x_k}^{l_k}}$, $k = 1,2,\dots,n,$, E_0 and E are Banach spaces, E_0 is continuously and densely embedded into E,

$$\|u\|_{W^l_{p,\gamma}(\mathbb{R}^n;E_0,E)} = \|u\|_{L_{p,\gamma}(\mathbb{R}^n;E_0)} + \sum_{k=1}^{n} \left\|D_k^{l_k} u\right\|_{L_{p,\gamma}(\mathbb{R}^n;E)} < \infty, 1 \le p < \infty.$$

3. Infinite system of Cauchy problem for convolution-differential equations

The property of maximal regularity for elliptic boundary-value problems was studied, for example in [2], [7], [17]. And for differential-operator equations in Banach spaces in [1], [6]. [18]. First we consider the convolution differential-operator equations

$$(L+\lambda)u = \sum_{k=0}^{l} a_k * \frac{d^k u}{dx^k} + A_\lambda * u = f(x)\,, x \in \mathbb{R} \tag{4}$$

in $L_{p,\gamma}(\mathbb{R}; E)$, where $A_\lambda = A + \lambda, A = A(x)$ is a possible unbounded operator in a Banach space E, $a_k = a_k(x)$ are complex valued functions on $\mathbb{R}$.

In [11-13] under certain conditions, it was proved that for equation (4) there exists a unique solution and the following coercive uniform estimate holds

$$\sum_{k=0}^{l} |\lambda|^{1-\frac{k}{l}} \left\|a_k * \frac{d^k u}{dx^k}\right\|_{L_{p,\gamma}(\mathbb{R};E)} + \|A * u\|_{L_{p,\gamma}(\mathbb{R};E)} + |\lambda| \|u\|_{L_{p,\gamma}(\mathbb{R};E)} \le C\|f\|_{L_{p,\gamma}(\mathbb{R};E),} \tag{5}$$

for $\lambda \in S_\varphi, f \in L_{p,\gamma}(\mathbb{R}; E), p \in (1, \infty), \varphi \in \pi)$.

Let L be an operator in $L_{p,\gamma}(\mathbb{R}; E)$generated by problem (4) for $\lambda = 0$, i.e.

$$D(L) \subset W^l_{p,\gamma}(\mathbb{R}; E(A), E), Lu = \sum_{k=0}^{l} a_k * \frac{d^k u}{dx^k} + A * u.$$

Again in [11-13], similarly, in the corresponding conditions it is proved that

$$\sum_{k=0}^{l} |\lambda|^{1-\frac{k}{l}} \left\|a_k * \frac{d^k}{dx^k}(L+\lambda)^{-1}\right\|_{\mathcal{L}(L_{p,\gamma}(\mathbb{R};E))} + \|A * (L+\lambda)^{-1}\|_{\mathcal{L}(L_{p,\gamma}(\mathbb{R};E))} + \|\lambda(L+\lambda)^{-1}\|_{\mathcal{L}(L_{p,\gamma}(\mathbb{R};E))} \le C \tag{6}$$

In this section, we establish the separability properties of problem (1) and maximal regularity of infinity system of Cauchy problem for parabolic CDEs (2). Further we prove that the operator generated by problem (1) is uniformly R-positive. Main tool of this section is the operator-valued Fourier multipliers. At the same time, coercive estimate for (1) is derived by using the representation formula for solution of problem (1) and operator valued multiplier results in $L_{p,\gamma}(\mathbb{R}; E)$.

Let $E = l_q$, B be a matrix such $B = \{b_{ij}\}$. By using these corollaries, we have the following results. First we define sufficient conditions that guarantee separability of problem (1).

Condition 3.1. Suppose the following are satisfied:

1. For

$$\xi \in \mathbb{R}\backslash\{0\}, L(\xi) = \sum_{k=0}^{l} \hat{a}_k(\xi)(i\xi)^k \in S_{\varphi_1}, \varphi_1 + \varphi < \pi,$$
$$L(\xi) \geq C|\hat{a}_k(\xi)||\xi^l|, a_k \in L_1(\mathbb{R}),$$

2. for $\{b_{ij}(x)\}_1^\infty \in l_q$, for all $x \in \mathbb{R}$, and some $x_0 \in \mathbb{R}$

$$C_1|b_{ij}(x_0)| \leq |b_{ij}(x)| \leq C_2|b_{ij}(x_0)|,$$

3. $\hat{a}_k, \hat{b}_{ij} \in C^{(1)}(\mathbb{R})$and $|\xi \hat{a}_k{}'(\xi)| \leq M_1, |\xi \hat{b}_{ij}{}'(\xi)| \leq M_2,$

where C_1, C_2, M_1, M_2 −are positive constants.

Let

$$a_{ij} = a_{ji}, \sum_{i,j=1}^{\infty} a_{ij}\xi_i\xi_j \geq C|\xi|^2, \ for\ \xi \neq 0, \ B(x) = b_{ij}(x),$$
$$b_{ij} > 0, u = \{u_j\}, \qquad B * u = \{b_{ij} * u_j\}$$

And

$$l_q(A) = \left\{u, u \in l_q, \|u\|_{l_{q(A)}} = \|B * u\|_{l_q} = \left(\sum_{i=1}^{\infty} |b_{ij}(x_0) * u_j|^q\right)^{1/q} < \infty\right\}.$$

Theorem 3.1. Suppose the Condition 3.1. satisfied, and $\gamma \in A_p$.Then for all $f(x) = \{f_i(x)\}_{i=1}^\infty \in L_{p,\gamma}(\mathbb{R}; l_q)$, for $\lambda \in S_\varphi$, the problem (1) has a unique solution $u = \{u_i\}_1^\infty$ that belongs to $W_{p,\gamma}^l(\mathbb{R}; l_q(B), l_q)$ and the following coercive uniform estimate holds:

$$\sum_{k=0}^{l} |\lambda|^{1-\frac{k}{l}} \left\|a_k * \frac{d^k u}{dx^k}\right\|_{L_{p,\gamma}(\mathbb{R};l_q)} + \|B * u\|_{L_{p,\gamma}(\mathbb{R};l_q)} + |\lambda|\|u\|_{L_{p,\gamma}(\mathbb{R};l_q)}$$
$$\leq C\|f\|_{L_{p,\gamma}(\mathbb{R};l_q)}, \tag{7}$$

and there exists a resolvent$(L + \lambda)^{-1}$of operator L and

$$\sum_{k=0}^{l} |\lambda|^{1-\frac{k}{l}} \left\|a_k * \frac{d^k}{dx^k}(L + \lambda)^{-1}\right\|_{\mathcal{L}(L_{p,\gamma}(\mathbb{R};l_q))}$$
$$+ \|B * (L + \lambda)^{-1}\|_{\mathcal{L}(L_{p,\gamma}(\mathbb{R};l_q))}$$
$$+ \|\lambda(L + \lambda)^{-1}\|_{\mathcal{L}(L_{p,\gamma}(\mathbb{R};l_q))} \leq C$$

Now, we consider the Cauchy problem for the following system of integro-differential equations of infinite orders.

$$\frac{\partial u_j}{\partial t} + \sum_{k=0}^{l} a_k * \frac{\partial^k u_i}{\partial x^k} + \sum_{j=1}^{\infty} b_{ij} * u_j(t,x) = f_i(t,x)$$

$$u_j(0,x) = 0, x \in \mathbb{R}, t \in \mathbb{R}_+. \tag{8}$$

Theorem 3.2. Let Condition 3.1. satisfied. The for all$f = \{f_i\}_1^\infty \in L_{\boldsymbol{p},\gamma}(\mathbb{R}_+^2; l_q)$, for $\lambda \in S_\varphi$ and $\gamma \in A_p$, the problem (8) has a unique solution $u = \{u_i\}_1^\infty \in W_{\boldsymbol{p},\gamma}^{1,l}(\mathbb{R}_+^2; l_q(B), l_q)$and the coercive estimate holds:

$$\left\|\frac{\partial u}{\partial t}\right\|_{L_{\boldsymbol{p},\gamma}(\mathbb{R}_+^2;l_q)} + \sum_{k=0}^{l} \left\|a_k * \frac{\partial^k u}{\partial x^k}\right\|_{L_{\boldsymbol{p},\gamma}(\mathbb{R}_+^2;l_q)} + \|B * u\|_{L_{\boldsymbol{p},\gamma}(\mathbb{R}_+^2;l_q)} \le C\|f\|_{L_{\boldsymbol{p},\gamma}(\mathbb{R}_+^2;l_q)}. \tag{9}$$

Proof: For this purpose will be denote the space of all $\boldsymbol{p} = (p, p_1)$ $-$summable E $-$valued functions with mixed norm through $L_{\boldsymbol{p},\gamma}(\mathbb{R}_+^2; l_q)$. So, $L_{\boldsymbol{p},\gamma}(\mathbb{R}_+^2; l_q)$, denote the space of all measurable E $-$valued functions defined on $\mathbb{R}_+^2$ with the norm

$$\|f\|_{L_{\boldsymbol{p},\gamma}(\mathbb{R}_+^2;l_q)} = \left(\int_{\mathbb{R}} \left(\int_{\mathbb{R}_+} \gamma(x)\, \|\{f_i(x)\}\|_{l_q}^p dx\right)^{p_1/p} dt\right)^{1/p_1} < \infty.$$

Respectively, we define $W_{p,\gamma}^{1,l}(\mathbb{R}_+^2; l_q(B), l_q)$.

Let H denote the differential operator in $L_{\boldsymbol{p},\gamma}(\mathbb{R}; l_q)$. generated by the following elliptic CDE

$$\sum_{k=0}^{l} a_k * \frac{\partial^k u}{\partial x^k} + B * u = f, \tag{10}$$

where $u = \{u_i(t,x)\}_1^\infty, f = \{f_i(t,x)\}_1^\infty, B * u = \{b_{ij} * u_j\}$.

First we prove R $-$positivity of the operator B. For the prove of R $-$positivity, we need to show the R $-$boundedness of the set $\{\lambda(B+\lambda)^{-1}; \lambda \in S_\varphi\}$.

$$\lambda(B+\lambda)^{-1} = \frac{\lambda}{\Delta\, B(\lambda)}[B_{ij} + \lambda],$$

where$\Delta\, B(\lambda) = det(B+\lambda), B_{ij}$ is the algebraic complement of the element of the matrix b_{ij} in its determinant. Then by definition of R $-$boundedness we obtain

$$\int_0^1 \left\|\sum_{k=1}^m r_k(y)\lambda_k\,(B+\lambda_k)u_k\right\|_{l_q}^q dy$$

$$\leq C\int_0^1 \sum_{k=1}^m \left|\sum_{i=1}^\infty\sum_{j=1}^\infty \frac{\lambda_k}{\triangle\,B(\lambda_k)}\left(B_{ij}+\lambda_k\right)r_k(y)u_k\right|_{l_q}^q dy$$

$$\leq \sup_{k,i}\sum_{j=1}^\infty \left|\frac{\lambda_k}{\triangle\,B(\lambda_k)}\left(B_{ij}+\lambda_k\right)\right|_{l_q}^q \int_0^1 \left|\sum_{k=1}^m r_k(y)u_k\right|_{l_q}^q dy$$

for all $u_1, u_2, \ldots, u_m \in l_q$, $\lambda_1, \lambda_2, \ldots, \lambda_m \in S_\varphi$, where $\{r_k\}$ is a sequence of independent symmetric $\{-1;1\}$ −valued random variables on [0,1]. Since B is symmetric and positive definite, it generates a positive operator in l_q, for $q \in (1,\infty)$, we have

$$\int_0^1 \left\|\sum_{k=1}^m r_k(y)\lambda_k\left(B_{ij}+\lambda_k\right)u_k\right\|_{l_q}^q dy \leq C\int_0^1 \left\|\sum_{k=1}^m r_k(y)\,u_k\right\|_{l_q}^q dy$$

So, from this we get that, the set $\{\lambda(B+\lambda);\lambda\in S_\varphi\}$ is R −bounded, i.e. the operator B is R −positive in l_q.

From (5) we obtain that problem (10) has a unique solution $u \in W_{\boldsymbol{p},\gamma}^{1,l}\left(\mathbb{R}_+^2; l_q(B), l_q\right)$for $f \in L_{\boldsymbol{p},\gamma}(\mathbb{R}_+^2; l_q)$ and the estimate (7) holds. It is known that, the problem (8) can be expressed as

$$\frac{\partial u_i(t)}{\partial t} + Hu_i(t) = f_i(t)–,$$
$$u(0) = 0, t \in \mathbb{R}_+ \qquad (11)$$

By using the additional and product properties of R −bounded operators [5,Proposition 3.4] and Kahane's contraction principle [5, Lemma 3.5] for family of R −bounded operators we have the R −positivity of operator H. Taking into account $L_{p,\gamma}(\mathbb{R};E)$ ∈UMD [see [2]) and R −positivity of operator H, for $\varphi \in (\frac{\pi}{2},\pi)$ we obtain that for $f \in L_{p_1}(\mathbb{R}_+; L_{p,\gamma}(\mathbb{R}; l_q))$ (see [5], [11]) the equation (11) has a unique solution $u \in W_{p_1}^1\left(\mathbb{R}_+; D(H), L_{\boldsymbol{p},\gamma}(\mathbb{R}; l_q)\right)$satisfying

$$\left\|\frac{\partial u}{\partial t}\right\|_{L_{p_1}(\mathbb{R}_+;L_{p,\gamma}(\mathbb{R};l_q))} + \|Hu\|_{L_{p_1}(\mathbb{R}_+;L_{p,\gamma}(\mathbb{R};l_q))}$$
$$\leq C\|f\|_{L_{p_1}\left(\mathbb{R}_+;L_{p,\gamma}(\mathbb{R};l_q)\right)} \qquad (12)$$

By virtue of estimate (7) and (8) from the (12) we get (9).

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Коэффициент оценки решений бесконечной системы интегро-дифференциальных уравнений в весовых пространствах

Г.К. Мусаев
Бакинский государственный университет, кафедра дифференциального и интегрального уравнений, Баку, Азербайджан
e-mail: gkm55@mail.ru

РЕЗЮМЕ

Изучаются некоторые свойства сверточных дифференциально-операторных уравнений в банаховом пространстве. Также установлено, что соответствующий оператор реализации положителен и R-положителен. Эти результаты позволяют показать отделимость дифференциальных операторов в E-значном взвешенном L_p-пространстве. В настоящей работе получены свойства положительности и сепарабельности сверто-эллиптических операторов бесконечных систем интегро-дифференциальных уравнений.

Ключевые слова: Бесконечная система, интегро-дифференциальные уравнения, R-положительность, взвешенное условие мультипликатора, уравнения свертки.