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THE $R-$ INDEX OF SOME GRAPHS

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Abstract. Topological indices, beginning with the classical Wiener index, play a very significant role in analysing the physio-chemical and biological properties of a chemical molecule under investigation through its graphical properties. The quantitative structure property relationship studies and quantitative structure activity studies are aided through the study of the mathematical properties of those topological indices. Avoluminous research have been done on those indices with respect to the classical degrees of vertices of a graph. In [3] one of the present authors generalises the concept of the degree of a vertex to the $R-$ degree and defined the $R-$index of graphs and obtained them in cases of a complete graph, paths and cycles. In this paper we obtain the $R-$ index of the subdivision graph of a regular graph, subdivision graph of a wheel, tadpole graph, Fan Graph, Gear Fan Graph and Gear Wheel Graph.

Keywords: R-degree, R-index.

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1. Introduction

Let G be a simple connected graph on n vertices and m edges. The degree of a vertex v, deg(v) is the number of vertices adjacent to v. Chemical graph theory and mathematical chemistry deals with the study of chemical and mathematical properties of a chemical molecule. In pharmaceutical sciences, the design and prediction of the activity of a drug can be mathematically studied using some invariants associated with its graph structure. These invariants are generally called topological indices and the quantitative structure property relationship (QSPR) studies and quantitative structure activity relation (QSAR) studies are aided through the study of the mathematical properties of them. Generally, topological indices show a good correlation with different physio-chemical properties of corresponding chemical compounds, so that now a days topological indices are used as a standard tool in studying several properties of chemical compounds. The first distance based topological index was proposed by Wiener in 1947 for modeling physical properties of alkanes, and after him, a very many topological indices were defined by chemists and mathematicians, which helped in studying properties of chemical structures in detail. In 1972 I. Gutman et.al proposed the first degree based topological indices namely the Zagreb indices. These topological indices were used to measure the branching of the carbon-atom skeletons. For a detailed information on degree based indices see [2, 3] and the references cited therein.

Ediz in [3] introduced the concept of $R-$degree of a vertex and defined some $R-$indices in tune with the existing degree based topological indices and computed them for complete graphs, paths and cycles and for some recent works see [3, 4, 5] and the references cited therein. In this paper we obtain the $R-$ index of the subdivision graph of a regular graph, subdivision graph of a wheel, tadpole graph, Fan Graph, Gear Fan Graph and Gear Wheel Graph. For basic graph theoretic terminology, see [1].

2. R-degree and R-indices of graphs

Let G be a graph with vertex set $V(G)$ and $u \in V(G)$. Then the sum of the degrees of vertices adjacent to u is denoted by $s(u)$ and the product of the degrees of vertices adjacent to u by $m(u)$.

Definition 1. *[3] Let G be a graph and $u \in V(G)$. Then the r-degree of u denoted by $r(u)$ is defined as*

$r(u) = s(u) + m(u)$.

Definition 2. *[3] The first $R-$ index $R^1(G)$ is defined as*

$R^1(G) = \sum_{u \in V(G)} r(u)^2$

Definition 3. *[3] The second $R-$ index $R^2(G)$ is defined as*

$$R^2(G) = \sum_{uv \in E(G)} r(u)r(v)$$

Definition 4. *[3] The second $R-$ index $R^3(G)$ is defined as*

$$R^3(G) = \sum_{uv \in E(G)} (r(u) + r(v))$$

3. R indices of the subdivision graph of a regular graph

Theorem 1. *Let G be a (p,q) k-regular graph and S(G) be its subdivision graph. Then*

1. $R^1(S(G)) = p(2^{2k} + 4k \times 2^k + 2k^4 + 2k^3 + 4k^2 + \frac{k^5}{2})$
2. $R^2(S(G)) = pk(2^k + 2k)(k^2 + 2k)$

3 $R^3(S(G)) = pk(2^k + k^2 + 4k)$

Proof. Let $v_1, v_2, \ldots, v_p$ be the vertices of G and $u_1, u_2, \ldots, u_q$ be the vertices corresponding to the edges of G. Then each v_i, $i = 1, 2, \ldots, p$ is adjacent to k vertices of degree 2 corresponding to the k edges incident with v_i. Also each u_j, $j = 1, 2, \ldots, q$ is adjacent with two vertices of degree k. Thus in $S(G)$ we have the following partition of vertices with respect to $s(u)$ and $m(u)$. Recall that $S(G)$ has $p + q$ vertices and pk edges

vertex, u	$s(u)$	$m(u)$	$r(u)$	Number of vertices
v_i, $i = 1, 2, ..p$	2k	2^k	$2^k + 2k$	p
u_j, $j = 1, 2, ..., q$	2k	k^2	$k^2 + 2k$	q

Then by definition

$$R^1(G) = \sum_{u \in S(G)} r(u)^2 = \sum_{v_i} r(v_i)^2 + \sum_{u_j} r(u_j)^2$$

$$= p(2^k + 2k)^2 + q(k^2 + 2k)^2$$

$$= p(2^{2k} + 4k2^k + 2k^4 + 2k^3 + 4k^2 + \frac{k^5}{2})$$

Since the edges of S(G) are of the form $v_i u_j$, We have the following

$$R^2(S(G)) = \sum_{v_i u_j \in E(S(G))} (r(v_i) r(u_j)) = pk(2^k + 2k)(k^2 + 2k)$$

And

$$R^3(S(G)) = \sum_{v_i u_j \in E(S(G))} (r(v_i) + r(u_j)) = pk((2^k + 2k) + (k^2 + 2k))$$

$$= pk(2^k + k^2 + 4k)$$

4. R index of subdivision graph of wheel graph

Theorem 2. *Let $S(W_n)$ denote the subdivision graph of the Wheel W_n. Then*

1 $R^1(S(W_n)) = (2(n-1) + 2^{n-1})^2 + (n-1)[(4n-1)^2 + 421]$

2 $R^2(S(W_n)) = (n-1)[(4n-1)\{2^{n-1} + 2(n-1) + 14\} + 420]$

3 $R^3(S(W_n)) = (n-1)(2^{n-1} + 10n + 68)$

Proof. We label the vertices of $S(W_n)$ as follows. Let $v_1, v_2, \ldots, v_{n-1}$ denote the vertices of the cycle C_{n-1} and v_n denote that of K_1 in $W_n = C_{n-1} + K_1$. Let w_i, $i = 1, 2, .., n-1$ be the vertices used to subdivide the edges v_iv_{i+1} and u_j, $j = 1, 2, .., n-1$ be the vertices used to subdivide the edges v_nv_j in order. We observe that each v_i is adjacent with three vertices of degree 2, each w_i is adjacent with two vertices of degree 3, each u_i is adjacent with one vertex of degree 3 and one vertex of degree $n-1$ and v_n is adjacent with $n-1$ vertices of degree 2 for $i = 1, 2, \ldots, n-1$. Then we have the following partition of vertices based on $R-$degree of vertices. Note that $S(W_n)$ has $3n-2$ vertices and $4(n-1)$ edges.

vertex, u	$r(u)$	Number of vertices
v_i	14	$n-1$
v_n	$2^{n-1} + 2(n-1)$	1
w_i	15	$n-1$
u_i	$4n-1$	$n-1$

The result follows from a simple computation.

5. R index of the Tadpole graph $T_{n,k}$

The Tadpole graph $T_{n,k}$ is obtained by joining an end vertex of a path P_k to a vertex of the cycle C_n by an edge. Suppose we label the vertices of C_n as $v_1, v_2, \ldots, v_n$ and P_k be the path $u_1u_2 \ldots u_k$ and v_1 is joined to u_1 to get $T_{n,k}$. We observe that v_1 is of degree 3 and is adjacent with three vertices of degree 2, v_2, v_n and u_1 are adjacent with a vertex of degree 3 and one vertex of degree 2 each, u_k is adjacent with a vertex of degree 2 and all other vertices are adjacent with two vertices of degree 2. Also note that $T_{n,k}$ has $n+k$ vertices and $n+k$ edges. Then we have the following partition of vertices based on $r-$ degree of vertices.

vertex,u	$r(u)$	Number of vertices
v_1	14	1
v_2, v_n, u_1	11	3
$v_i, u_j, i = 3,4,\ldots,n-1; j = 2,3,\ldots,k-2$	8	$n+k-6$

u_{k-1}	5	1
u_k	4	1

Then from the above table we have the following theorem

Theorem 3. *Let $T_{n,k}$ be the tadpole graph on n+k vertices. Then*

1 $R^1(\mathrm{T}_{n,k}) = 64(n+k) - 216$

2 $R^2(\mathrm{T}_{n,k}) = 64(n+k) - 274$

3 $R^3(\mathrm{T}_{n,k}) = 16(n+k) - 26$

6. R index of the Fan Graph F_n

Let P_n be a path with n vertices, then the Fan Graph $F_n = \{u\} \vee P_n$ where $\vee$ denotes the join of graphs. We label the vertices of P_n as v_1, v_2, .., v_n. We observe that the vertex u of F_n is of degree n and the end vertices v_1, v_n of P_n is of degree 2 and all other vertices $v_i, i \neq 1, n$ is of degree 3 in F_n. Also note that F_n has $n+1$ vertices and $2n-1$ edges. Then we have the following partition of vertices based on the r−degree of vertices.

vertex, u	$s(u)$	$m(u)$	$r(u)$	Number of vertices
v_i , $i = 1, n$	$n+3$	$3n$	$4n+3$	2
v_i , $i = 2, n-1$	$n+5$	$6n$	$7n+5$	2
$v_i, i \neq 1,2,n-1,n \in P_n$	$n+6$	$9n$	$10n+6$	$n-4$
u	$3n-2$	$4 \times 3^{n-2}$	$3n-2+4\times 3^{n-2}$	1

Theorem 4. *Let F_n be the Fan graph with n+1 vertices. Then*

1. $R^1(F_n) = 100n^3 - 141n^2 - 268n - 72 + (24n-16)3^{n-2} - 16\times 3^{2n-4}$
2. $R^2(F_n) = 100n^3 - 184n^2 - 298n - 90 + (3n-2+4\times 3^{n-2})(10n^2 - 12n - 8)$
3. $R^3(F_n) = 33n^2 - 58n - 30 + 4n\times 3^{n-2}$

7. R index of the Gear Fan Graph $\overline{F}_n$

Let P_n be a path with n vertices, then the Fan Graph is $F_n = \{u\} \vee P_n$. Subdividing every edge of the fan path P_n of fan graph F_n, results in the gear fan graph denoted by $\overline{F_n}$. We label the vertices of P_n as v_1, v_2, .., v_n and the new vertices obtained from subdividing the edges of P_n are denoted as u_1, u_2, , u_{n-1}. We observe that the vertex u of $\overline{F_n}$ is of degree n and the end vertices v_1, v_n of P_n is of degree 2 and all other vertices v_i, $i \neq 1, n$ is of degree 3 in $\overline{F_n}$. Each new vertex u_j, $j = 1, 2, .., n-1$ obtained by subdivision of edges of P_n is of degree 2. Also note that $\overline{F} n$ has $2n$ vertices and $3n - 2$ edges. Then we have the following partition of vertices based on r-degree of vertices

vertex, u	$s(u)$	$m(u)$	$r(u)$	Number of vertices
v_i , $i = 1, n$	$n + 2$	$2n$	$3n + 2$	2
v_i , $i \neq 1, n$	$n + 4$	$4n$	$5n + 4$	$n - 2$
u_i , $i = 1, n - 1$	5	6	11	2
u_i , $i \neq 1, n - 1$	6	9	15	$n - 3$
u	$3n - 2$	$4 \times 3^{n-2}$	$3n - 2 + 4 \times 3^{n-2}$	1

Theorem 5. *Let $\overline{F_n}$ be the Gear Fan graph with 2n vertices. Then*

1 $R^1(\overline{F_n}) = 25n^3 + 17n^2 + 173n - 453 + (6n-4)4 \times 3^{n-2} + 16 \times 3^{2n-4}$

2 $R^2(\overline{F_n}) = 15n^3 + 80n^2 - 72n - 64 + (4 \times 3^{n-2})(5n^2 - 6)$

3 $R^3(\overline{F_n}) = 14n^2 + 26n - 46 + 4n \times 3^{n-2}$

8. R index of the Gear Wheel Graph $\overline{W_n}$

Let C_n be a cycle with n vertices, then the Wheel Graph is $W_n = \{u\} \vee C_n$. Subdividing every edge of the wheel cycle C_n of wheel graph W_n, results in the gear wheel graph denoted by $\overline{W_n}$. We label the vertices of C_n as v_1, v_2, .., v_n and the new vertices obtained from subdivision of edges of C_n as u_1, u_2, , u_n. We observe that the vertex u of $\overline{C_n}$ is of degree n and the vertices $v_i, i = 1, 2, .., n$ of C_n is of degree 3. Each new vertex u_j, $j = 1, 2, ..., n$ obtained by subdivision of edges of C_n is of degree 2. Also note that $\overline{W_n}$ has $2n+1$ vertices and $3n$ edges. Then we have the following partition of vertices based on the $r-$ degree of vertices.

vertex, u	$s(u)$	$m(u)$	$r(u)$	Number of vertices
v_i , i = 1, 2, ..., n	n +4	4n	5n +4	n
u_i , i = 1, 2, ..., n	6	9	15	n
u	3n	3^n	$3n + 3^n \times 3^{n-2}$	1

Theorem 6. *Let $\overline{W_n}$ be the Gear Wheel graph with 2n+1 vertices. Then*

1 $R^1(\overline{W_n}) = 25n^3 + 49n^2 + 241n + 3^{2n} + 6n \times 3^n$

2 $R^2(\overline{W_n}) = 15n^3 + 162n^2 + 120n + 5n^2 \times 3^n + 4n \times 3^n$

3 $R^3(\overline{W_n}) = 18n^2 + 42n + n \times 3^n$

Conclusion: In this paper a recently introduced index is computed for certain classes of graphs. This study can be proceeded by analyzing the relation between the R index of graph operations, computing the index of various graph operations and studying the chemical relevance of this index in case of graphs related to chemical compounds.

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