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A NUMERICAL SOLUTION OF THE MODIFIED BURGERS EQUATION USING LINEARIZED FINITE DIFFERENCE METHODS

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Abstract. In this article, we propose finite difference methods based on two different linearization techniques in order to obtain numerical solutions of the modified Burgers equation. The application of Fourier stability analysis method and computation of the error norms L_2 and L_∞ have shown that the results obtained from the computer runs are compatible with the relevant ones in the literature. A numerical example has been used to show the application and accuracy of the proposed method in comparison to other ones.

Keywords: Modified Burgers equation, finite difference method, Fourier stability analysis.

AMS Subject Classification: 97N40, 65N30, 65M06.

1. Introduction

First of all, Bateman [1] has introduced and studied Burgers equation and also shown that the equation is worth to study. But after its first introduction by Bateman, the equation has been widely studied in detail by Burgers in [2,3] as a mathematical model for turbulence and therefore the equation has been widely known with his name. The equations has been so fundamental in engineering and other scientific fields that it has found many applications in such diverse fields of science as convection and diffusion, number theory, gas dynamics, heat conduction, elasticity etc. [17]. Today, it still preserves its place and importance in the literature due to its accurate modeling of many physical phenomena.
The one-dimensional generalized Burgers equation is given in the form of

$$u_t + u^p u_x - \nu u_{xx} = 0, \qquad a \le x \le b,\ t \ge 0$$

where u=u(x,t) is a function of the space and time variables respectively denoting the velocity, ν denotes a positive constant showing the kinematic viscosity of the fluid, and p denotes a positive parameter. If we choose p=1 and p=2 we get Burgers equation and modified Burgers equation, respectively.

In the past a few years, many authors have tried several schemes and techniques in order to obtain both the analytical and numerical solutions of the equation. Among others, Benton and Platzman [4] have obtained its analytical solution; Miller [miller] has derived infinite series solutions of the problem. As for the numerical ones, among many other studies conducted in the literature, Dag et

al. have used B-spline collocation methods for numerical solutions of the Burgers equation, (see [8] and the references therein).
In this paper, a variation form of the Burgers equation is going to be taken into consideration that is the modified Burgers equation, presented in the form of

$$u_t + u^2 u_x - \nu u_{xx} = 0, \qquad a \le x \le b, \tag{1}$$

here u(x,t) denotes the dependent variable, ν denotes the viscosity parameter, and t and x denote the independent parameters, namely time and space, respectively.

In this paper we aim to apply the finite difference methods in order to build a numerical method for the numerical solutions of the modified Burgers equation. Several authors have tried to solve Eq. (1) both analytically and numerically using various schemes. As for the numerical one, among others available in the literature, Ramadan and El-Danaf [14] have solved the problem by using the collocation method with quintic splines. After that, the same equation has been numerically solved by Ramadan et al. [15] using the collocation method but now with septic splines. The Burgers and modified Burgers equations have been solved by Saka and Dag [18] with the application of time and space splitting techniques and then employed the quintic B-spline collocation procedure to approximate the resulting systems. Irk [11] has employed Crank-Nicolson central differencing scheme for the time integration and sextic B-spline functions for the space integration to the modified and time splitted modified Burgers equation. A numerical solution has been proposed by Temsah [19] for the convection-diffusion equation using El-Gendi method with interface points and then numerical results for Burgers and modified Burgers equations have been shown. Grienwank and El-Danaf [12] have proposed a non-polynomial spline based method to obtain numerical solutions of the non-linear modified Burgers equation. Bratsos [5] has used a finite-difference scheme based on rational approximations to the matrix-exponential term in a two-time level recurrence relation for the numerical solution of the modified Burgers equation. Bratsos [6] has presented a finite-difference scheme based on fourth-order rational approximants to the matrix-exponential term in a two-time level recurrence relation for the numerical solution of the modified Burgers equation. Bratsos and Petrakis [7] have used an explicit finite difference scheme based on second-order rational approximations to the matrix-exponential term for the numerical solution of the modified Burgers equation. The equation has been numerically solved by Roshan and Bhamra [16] by the Petrov-Galerkin method using a linear hat function as the trial function and a cubic B-spline function as the test function.

During the solution process of the numerical example, the boundary conditions related to Eq. (1) are going to be

$$u(a,t) = \beta_1, \qquad u(b,t) = \beta_2, \qquad t \ge t_0.$$

The accuracy of the present method with the first and second linearization techniques is going to be tested using a numerical example and their stability analysis is investigated separately.

The layout of the present paper is as follow. In Section 2, the fundamentals of the finite difference method have been explained. In sections 3 and 4, two different linearization techniques have been applied to modified Burgers equations. In Section 5, the stability analysis of the method has been discussed. In Section 6, numerical examples have been presented to illustrate the application of the proposed method. The conclusion is given in Section 7 by summarizing the method and its numerical results.

2. The finite difference method

In order to obtain numerical solutions, the solution region of the problem R=[a,b]x[t_0] together with its boundary ∂R composed of points x=a and x=b and $t=t_0$ is covered by M equal subintervals of length h=(b-a)/M by the line $x_m=a+mh$, m=0 (1) M and time $t\geq t_0$ is incremented in steps of size k by the line $t_n=t_0+nk$, n=0(1)N. For the numerical solution of Eq. (1) at a given mesh point (mh , nk) is going to be replaced by u(mh,nk) and its approximating difference scheme is going to be replaced by U_m^n .

Substituting the dependent variable and its derivatives with their approximated values by the finite difference approximation and then applying Eq. (1) at each mesh point of the region result in either a single explicit equation or a system of difference equations generally written in a matrix-vector form. However, when it is applied to non-linear problems, it generally results in non-linear system of equations and those systems might not be solved directly. Thus, to overcome this inconvenience an appropriate numerical algorithm is used to solve those systems.

Linearization I:
If we use the forward difference approximation in place of u_t and the weighted central difference approximation in place of u_{xx} in Eq. (1) at each mesh point denoted by (m,n+1),

$$u_t \cong \frac{U_m^{n+1}-U_m^n}{k},$$

and

$$u_{xx} \cong \frac{1}{h^2}\left(\theta\left(U_{m+1}^{n+1}-2U_m^{n+1}+U_{m-1}^{n+1}\right)+(1-\theta)\left(U_{m+1}^{n}-2U_m^{n}+U_{m-1}^{n}\right)\right),$$

respectively, and finally apply the following linearization technique in place of the non-linear term u^2u_x

$$u^2u_x \cong \left(\frac{U_m^n+U_{m+1}^n}{2}\right)^2\left(\theta\left(\frac{U_{m+1}^{n+1}+U_{m-1}^{n+1}}{2h}\right)+(1-\theta)\left(\frac{U_{m+1}^{n}+U_{m-1}^{n}}{2h}\right)\right)$$

we simply result in the following system of algebraic equations

$$U_{m-1}^{n+1}\left(\theta\left(\frac{-h\left(U_m^n+U_{m+1}^n\right)^2-8\nu}{8h^2}\right)\right)+U_m^{n+1}\left(\theta\left(\frac{h^2+2k\nu\theta}{kh^2}\right)\right)+$$
$$U_{m+1}^{n+1}\left(\theta\left(\frac{h\left(U_m^n+U_{m+1}^n\right)^2-8\nu}{8h^2}\right)\right)=U_{m-1}^n\left((1-\theta)\left(\frac{h\left(U_m^n+U_{m+1}^n\right)^2+8\nu}{8h^2}\right)\right)+ \qquad (2)$$
$$U_m^n\left(\theta\left(\frac{h^2-2k\nu(1-\theta)}{kh^2}\right)\right)+U_{m+1}^n\left((1-\theta)\left(\frac{-h\left(U_m^n+U_{m+1}^n\right)^2+8\nu}{8h^2}\right)\right)$$

The Eq. (2) will be solved with the usage of a proper algorithm for those values of θ (θ=0,1/2,1).

Linearization II:

If we use the forward difference approximation in place of u_t and the weighted central difference approximation in place of u_{xx} in Eq. (1) at each mesh point denoted by (m,n+1),

$$u_t \cong \frac{U_m^{n+1}-U_m^n}{k},$$

and

$$u_{xx} \cong \frac{1}{h^2}\left(\theta\left(U_{m+1}^{n+1}-2U_m^{n+1}+U_{m-1}^{n+1}\right)+(1-\theta)\left(U_{m+1}^n-2U_m^n+U_{m-1}^n\right)\right),$$

respectively, and then apply the following linearization technique in place of the non-linear term u^2u_x

$$u^2u_x \cong \left(\frac{U_{m-1}^n+U_m^n}{2}\right)^2\left(\theta\left(\frac{U_{m+1}^{n+1}-U_{m-1}^{n+1}}{2h}\right)+(1-\theta)\left(\frac{U_{m+1}^n+U_{m-1}^n}{2h}\right)\right)$$

we simply result in the following system of algebraic equations

$$U_{m-1}^{n+1}\left(\theta\left(\frac{-h\left(U_{m-1}^n+U_m^n\right)^2-8\nu}{8h^2}\right)\right)+U_m^{n+1}\left(\frac{h^2+2k\nu\theta}{kh^2}\right)+$$
$$U_{m+1}^{n+1}\left(\theta\left(\frac{h\left(U_{m-1}^n+U_m^n\right)^2-8\nu}{8h^2}\right)\right)=U_{m-1}^n\left((1-\theta)\left(\frac{h\left(U_{m-1}^n+U_m^n\right)^2+8\nu}{8h^2}\right)\right)+ \qquad (3)$$
$$U_m^n\left(\theta\left(\frac{h^2-2k\nu(1-\theta)}{kh^2}\right)\right)+U_{m+1}^n\left((1-\theta)\left(\frac{-h\left(U_{m-1}^n+U_m^n\right)^2+8\nu}{8h^2}\right)\right).$$

The Eq. (3) will be solved using a proper algorithm for those values of θ (θ=0,1/2,1).

3. Stability analysis

Following the Fourier method of analyzing stability and using ξ^q as the amplication factor, the growth factor of a typical Fourier mode can be described as:

$$U_m^n = e^{i\beta ph}\xi^q \tag{4}$$

where $i=\sqrt{-1}$. To proceed inspecting the stability of the numerical scheme, the nonlinear term u^2u_x in the modified Burgers equation is linearized by assuming the quantity u^2 as a local constant. In that case, the nonlinear term in the equation changes into $\hat{U}u_x$ and thus the Eq. (1) becomes

$$u_t + \hat{U}u_x - \nu u_{xx} = 0 .$$

If the weighted average approximation is taken as follows

$$\begin{aligned}&\left(\frac{U_m^{n+1}-U_m^n}{k}\right)+\hat{U}\left(\theta\left(\frac{U_{m+1}^{n+1}-U_{m-1}^{n+1}}{2h}\right)+(1-\theta)\left(\frac{U_{m+1}^{n}-U_{m-1}^{n}}{2h}\right)\right)-\\ &\frac{\nu}{h^2}\left(\theta\left(U_{m+1}^{n+1}-2U_m^{n+1}+U_{m-1}^{n+1}\right)+(1-\theta)\left(U_{m+1}^{n}-2U_m^{n}+U_{m-1}^{n}\right)\right)=0,\end{aligned} \tag{5}$$

then the generalized m^{th} row of Eq. (5) becomes

$$\begin{aligned}&U_{m-1}^{n+1}\left(-\frac{\theta\hat{U}}{2h}-\frac{\nu\theta}{h^2}\right)+U_m^{n+1}\left(\frac{1}{k}+\frac{2\nu\theta}{h^2}\right)+U_{m+1}^{n+1}\left(\frac{\theta\hat{U}}{2h}-\frac{\nu\theta}{h^2}\right)=\\ &U_{m-1}^{n}\left(\frac{(1-\theta)\hat{U}}{2h}+\frac{\nu(1-\theta)}{h^2}\right)+U_m^{n}\left(\frac{1}{k}-\frac{2\nu(1-\theta)}{h^2}\right)+\\ &U_{m+1}^{n}\left(-\frac{(1-\theta)\hat{U}}{2h}+\frac{\nu(1-\theta)}{h^2}\right).\end{aligned} \tag{6}$$

If we substitute the Fourier mode (4) into this linearized recursive relationship, then (6) yields

$$g=\frac{a-ib}{c-id}\ , \tag{7}$$

where

$$\begin{aligned}a&=h^2-2k\nu+2k\nu\theta-2k\nu(\theta-1)\cos\phi,\\ b&=-kh\hat{U}(\theta-1)\sin\phi,\\ c&=h^2+2k\nu\theta-2k\nu\theta\cos\phi,\\ d&=-kh\hat{U}\theta\sin\phi.\end{aligned} \tag{8}$$

It is time to investigate three different conditions of θ. If we take:

- $\theta=0$, then it coincides with explicit method, and it is required to satisfy the following inequality for the stability condition
$$h^4-(h^2-2k\nu+2k\nu\cos\phi)^2-h^2k\hat{U}^2\sin^2\phi\geq 0$$
- $\theta=1$, then it corresponds to implicit method, and it is required to satisfy the following inequality for the system to be stable
$$4h^2k\nu+4k^2\nu^2-4h^2k\nu\cos\phi-8k^2\nu^2\cos\phi+4k^2\nu^2\cos^2\phi+h^2k^2\hat{U}^2\sin^2\phi\geq 0$$
- $\theta=1/2$, then it coincides with Crank-Nicolson method, and the scheme is seen to be unconditionally stable by the following inequality
$$-4h^2k\nu(\cos\phi-1)\geq 0$$

After some rudimentary arithmetic operations, we see that the stability condition $|g|\leq 1$ is satisfied by the following inequality:
$$c^2+d^2-a^2-b^2=96h^2\nu\Delta t(2+\cos\phi)\sin\left[\frac{\phi}{2}\right]^2\geq 0$$
thus we conclude that the linearized scheme is unconditionally stable.

4. Numerical results

In order to test the applicability of the present method, a test problem has been used in the present study, and the numerical results of the equation have been obtained and all computations have been run on a Pentium i7 PC in the FORTRAN code using double precision arithmetic. To show the accuracy of the results, both the error norm L_2
$$L_2=\|u-U_N\|=\sqrt{h\sum_{j=0}^{N}\left|u_j-(U_N)_j\right|^2},$$
and the error norm L_∞
$$L_\infty=\|u-U_N\|_\infty=\max_j\left|u_j-(U_N)_j\right|.$$
have been calculated and presented.

The exact solution of the modified Burgers equation is as follows [9]
$$u(x,t)=\frac{x/t}{1+\sqrt{t}/c_0\exp(x^2/4\nu t)},\quad t\geq t_0,\quad 0\leq x\leq 1$$
where c_0 is a constant, $0< c_0 <1$ and $t_0=1$.

During numerical computations, for the numerical solution of the test problem two different linearization techniques have been applied. For reasons of comparison with the relevant results in [11,14,15] the values of the error norms L_2 and L_∞ have been computed at times t=2,6,10 for different values of h,ν and Δt and $c_0=0.5$. Table 1 compares the error norms L2 and L_∞ of the present study with those of other studies for h=0.005, ν=0.01 and Δt=0.01 at times t=2, 6 and 10. In order to see the behavior of the wave in a wider range, the solution domain of the test problem has also been taken as [0, 1.3]. Table 2 compares the error norms L_2 and L_∞ of the present study with those of other studies for h=0.005, ν=0.005 and Δt=0.010 at the same time levels . While Table 3 compares the error norms L_2 and L_∞ of the present study with those of other studies for h=0.005, ν=0.001 and Δt=0.01, Table 4 makes a comparison of the same error norms for h=0.02, ν=0.01 and Δt=0.01 at times t=2,6 and 10. As seen from the tables, the obtained results using both of the two linearization techniques are in good agreement with those available in the literature.

Since the graphs of the numerical solutions obtained using Linearization I and Linearization II would be indiscriminately similar to each other, we have presented the graphs of Linearization I. The computed numerical results together with their errors are graphed in Figures 1 - 3 for various values of ν at different time levels of Linearization for θ=1/2 . But the graphs of the errors have only been drawn at time t=10. It can be seen that the maximum error happens at the right-hand boundary of the solution domain for ν=0.01. However, the errors for ν=0.005 and ν=0.001 have been recorded around the points where the waves get their highest amplitudes. The solution profiles clearly show that the computed solutions display correct physical behavior for different values of t.

One can easily conclude from the tables and graphs that the linearization schemes used in the present paper produce as good as or better results found in the related articles. Due to the simplicity of the method and its applicability to computer software, the method results in much better solutions in relatively low costs in terms of time and space. The figures drawn at different time levels and the tables comparing the results of both the present and related articles show the accuracy of the linearization schemes.

Table1. Comparison of the error norms L_2 and L_∞ with those in other studies in the literature at t=2, 6, 10 for h=0.005, Δt=0.01 and ν=0.01.

	$t=2$		$t=6$		$t=10$	
	$L_2 \times 10^3$	$L_\infty \times 10^3$	$L_2 \times 10^3$	$L_\infty \times 10^3$	$L_2 \times 10^3$	$L_\infty \times 10^3$
Lin. I (θ=0, Δt=0.001)	0.37876	0.81658	0.32614	0.52579	0.54709	1.28125
Lin. I (θ=1)	0.39173	0.83206	0.32699	0.52579	0.54720	1.28125
Lin. I (θ=1/2)	0.37988	0.81793	0.32621	0.52579	0.54711	1.28125
Lin. II (θ=0, Δt=0.001)	0.37860	0.81558	0.32585	0.52579	0.54694	1.28125
Lin. II (θ=1)	0.39160	0.83109	0.32670	0.52579	0.54705	1.28125
Lin. II (θ=1/2)	0.37972	0.81693	0.32592	0.52579	0.54695	1.28125
[10]	0.52308	1.21698	0.49023	0.72249	0.64007	1.28124

[11]	0.79043	1.70309	0.57672	0.76105	0.80026	1.80329
[13],(SBCM1)	0.38489	0.82934	-	-	0.54826	1.28127
[13], (SBCM2)	0.39078	0.82734	-	-	0.54612	1.28127
Lin. I (θ=0, Δt=0.001),[0,1.3]	0.37876	0.81658	0.27641	0.46536	0.25408	0.32464
Lin. I (θ=1), [0,1.3]	0.39173	0.83206	0.27683	0.46760	0.25404	0.32517
Lin. I (θ=1/2), [0,1.3]	0.37988	0.81793	0.27645	0.46556	0.25408	0.32470
Lin. II (θ=0, Δt=0.001), [0,1.3]	0.37860	0.81558	0.27609	0.46487	0.25385	0.32433
Lin. II (θ=1), [0,1.3]	0.39160	0.83109	0.27651	0.46712	0.25380	0.32485
Lin. II (θ=1/2), [0,1.3]	0.37972	0.81693	0.27612	0.46506	0.25385	0.32438
[13], (SBCM1), [0,1.3]	0.38489	0.82934	-	-	0.25586	0.32723
[13], (SBCM2), [0,1.3]	0.39078	0.82734	-	-	0.25259	0.32337

Table 2. Comparison of the error norms L_2 and L_∞ with those in other studies in the literature at t=2, 6, 10 for h=0.005, Δt=0.001 and ν=0.005.

	$t=2$		$t=6$		$t=10$	
	$L_2 \times 10^3$	$L_\infty \times 10^3$	$L_2 \times 10^3$	$L_\infty \times 10^3$	$L_2 \times 10^3$	$L_\infty \times 10^3$
Lin. I (θ=0)	0.22647	0.58022	0.16470	0.33003	0.13967	0.22897
Lin. I (θ=1)	0.22785	0.58228	0.16474	0.33032	0.13965	0.22902
Lin. I (θ=1/2)	0.22716	0.58125	0.16472	0.33017	0.13966	0.22899
Lin. II (θ=0)	0.22638	0.57952	0.16450	0.32969	0.13953	0.22874
Lin. II (θ=1)	0.22776	0.58159	0.16454	0.32997	0.13950	0.2288
Lin. II (θ=1/2)	0.22707	0.58055	0.16452	0.32983	0.13951	0.22877
[10]	0.25786	0.72264	0.22569	0.43082	0.18735	0.30006
[13], (SBCM1)	0.22890	0.58623	-	-	0.14042	0.23019
[13], (SBCM2)	0.23397	0.58424	-	-	0.13747	0.22626

Table 3. Comparison of the error norms L_2 and L_∞ with those in other studies in the literature at t=2, 6, 10 for h=0.005, Δt=0.01 and ν=0.001.

	$t=2$		$t=6$		$t=10$	
	$L_2 \times 10^3$	$L_\infty \times 10^3$	$L_2 \times 10^3$	$L_\infty \times 10^3$	$L_2 \times 10^3$	$L_\infty \times 10^3$
Lin. I (θ=0)	0.06696	0.25843	0.04942	0.14780	0.04072	0.10262

Lin. I (θ=1)	0.07115	0.26758	0.04954	0.14903	0.04063	0.10282
Lin. I (θ=1/2)	0.06901	0.26300	0.04947	0.14841	0.04067	0.10272
Lin. II (θ=0)	0.06693	0.25812	0.04936	0.14765	0.04067	0.10252
Lin. II (θ=1)	0.07112	0.26728	0.04948	0.14888	0.04058	0.10272
Lin. II (θ=1/2)	0.06898	0.26270	0.04941	0.14826	0.04063	0.10262
[10]	0.06703	0.27967	0.06046	0.17176	0.05010	0.12129
[11]	0.18355	0.81862	0.08142	0.21348	0.05512	0.13943
[13], (SBCM1)	0.06843	0.26233	-	-	0.04080	0.10295
[13], (SBCM2)	0.07220	0.25975	-	-	0.03871	0.09882

Table 4. Comparison of the error norms L_2 and L_∞ with those in other studies in the literature at t=2, 6, 10 for h=0.02, Δt=0.01 and ν=0.01.

	$t=2$		$t=6$		$t=10$	
	$L_2 \times 10^3$	$L_\infty \times 10^3$	$L_2 \times 10^3$	$L_\infty \times 10^3$	$L_2 \times 10^3$	$L_\infty \times 10^3$
Lin. I (θ=0)	0.37583	0.81044	0.32969	0.52579	0.55871	1.28125
Lin. I (θ=1)	0.39957	0.84143	0.33123	0.52579	0.5589	1.28125
Lin. I (θ=1/2)	0.38746	0.82525	0.33042	0.52579	0.55880	1.28125
Lin. II (θ=0)	0.37517	0.80634	0.32854	0.52579	0.55812	1.28125
Lin. II (θ=1)	0.39911	0.83767	0.33011	0.52579	0.55832	1.28125
Lin. II (θ=1/2)	0.38690	0.82148	0.32928	0.52579	0.55821	1.28125
[11]	0.79043	1.70309	0.51672	0.76105	0.80026	1.80239
[13], (SBCM1)	0.38474	0.82611	-	-	0.55985	1.28127
[13], (SBCM2)	0.41321	0.81502	-	-	0.55095	1.28127

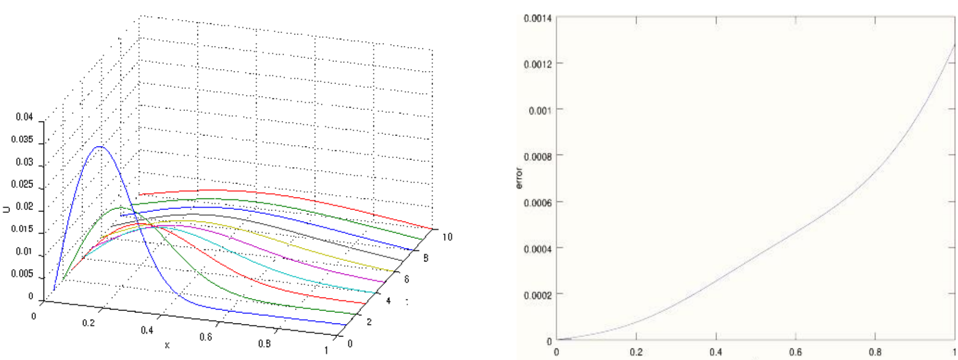

Figure 1. The numerical solutions of Problem at different times with ν = 0.01 using Linearization I for θ = 1/2, and error graph at t = 10.

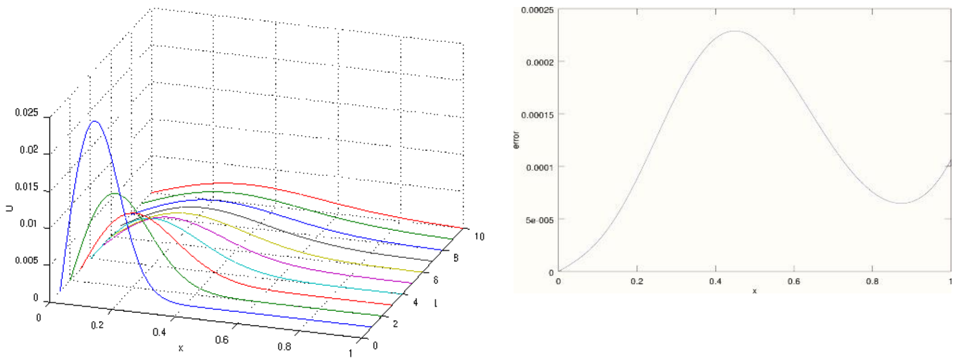

Figure 2. The numerical solutions of Problem at different times with v = 0.005 using Linearization I for θ = 1/2, and error graph at t = 10.

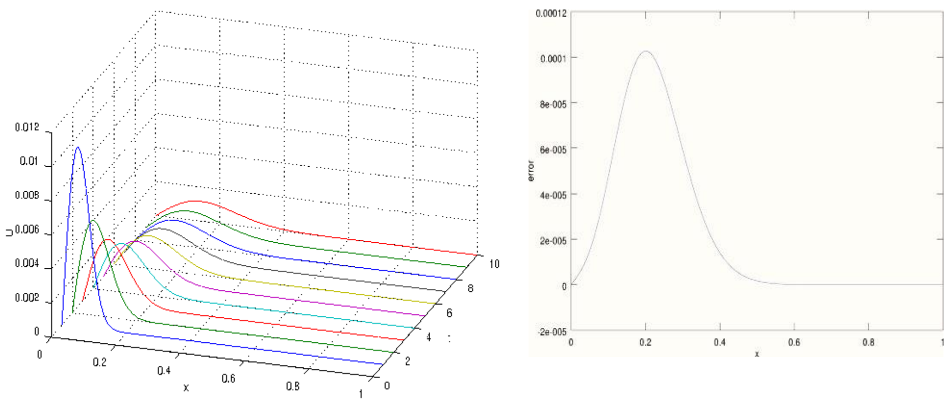

Figure 3. The numerical solutions of Problem at different times with v = 0.001 using Linearization I for θ = 1/2, and error graph at t = 10.

5. Conclusions

In this paper, a numerical treatment of the modified Burgers equation using two different linearization has been presented. To show the accuracy and efficiency of the presented method, it has been applied to a problem and a comparison has been made with the relevant ones in the literature. For error analysis, both of the error norms L2 and L_∞ have been computed and presented in tabular form. It has been seen from the obtained results that the error norms are sufficiently small during all computer runs. In conclusion, it can be said that the present method is a particularly successful numerical scheme for solving the Modified Burgers equation.

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Xətti sonlu fərqlər üsulunun köməyi ilə modifikasiya olunmuş Bürgers tənliyinin ədədi həlli

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XÜLASƏ

Məqalədə modifikasiya olunmuş Bürgers tənliyinin ədədi həllini almaq üçün iki fərqli xəttiləşdirmə üsuluna əsaslanan sonlu fərqlər üsulu təklif olunmuşdur. Furyenin dayanıqlı analiz üsulunun tətbiqi və xətaların normalarının hesablanması göstərir ki, kompüter hesablamalarından alınmış nəticələr ədəbiyyatdan götürülmüş uyğun nəticələrlə uzlaşır. Təklif olunan metodun digərləri ilə müqayisədə üstünlüyünü göstərmək üçün ədədi misaldan istifadə olunmuşdur.

Açar sözlər: modifikasiya olunmuş Bürgers tənliyi, sonlu fərqlər metodu, dayanıqlı Furye analizi.

Численное решение модифицированного уравнения Бюргерса с помощью линеаризованных конечных разностных методов

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РЕЗЮМЕ

В статье предложены конечные разностные методы, основанные на двух различных методах линеаризации для получения численного решения модифицированного уравнения Бюргерса. Применение метода анализа устойчивости Фурье и вычисление норм ошибок показали, что результаты, полученные из компьютерных вычислений совместимы с соответствующими им результатами в литературе. Числовой пример был использован, чтобы показать применение и точность предлагаемого способа по сравнению с другими.

Ключевые слова: модифицированное уравнение Бюргерса, метод конечных разностей, анализ устойчивости Фурье.