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THE TRIPLE LACUNARY STATISTICAL CONVERGENCE ON Γ^3 OVER p -METRIC SPACES DEFINED BY ORLICZ FUNCTION

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Abstract. In this paper, we define and study the notion of the triple lacunary statistical convergence and triple lacunary of statistical Cauchy sequences in random on Γ^3 over p - metric spaces defined by Orlicz function and prove some theorems which generalizes the results.

Keywords: analytic sequence, triple sequences, Γ^3 space, Musielak - Orlicz function, Random p-metric space, lacunary sequence, statistical convergence.

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1. Introduction

Throughout w, Γ and Λ denote the classes of all, entire and analytic scalar valued single sequences, respectively. We write w^3 for the set of all complex triple sequences (x_{mnk}), where $m,n,k \in N$, the set of positive integers. Then, w^3 is a linear space under the co-ordinate wise addition and scalar multiplication.
We can represent triple sequences by matrix. In case of double sequences we write in the form of a square. In the case of a triple sequence it will be in the form of a box in three dimensional case.

Some initial work on double series is found in Apostol [1] and double sequence spaces is found in Hardy [7], Subramanian et al. [2,18,23-27], and many others. Later on investigated by some initial work on triple sequence spaces is found in Sahiner et al. [10], Esi et al. [3-6,11], Subramanian et al. [12-22] and many others [28-29]. Let (x_{mnk}) be a triple sequence of real or complex numbers. Then the series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is called a triple series. The triple series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ give one space is said to be convergent if and only if the triple sequence (Smnk) is convergent, where

$$S_{mnk} = \sum_{i,j,q}^{m,n,k} x_{ijq} (m,n,k = 1,2,3,...).$$

A sequence $x = (x_{mnk})$ is said to be triple analytic if

$$sup_{m,n,k} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

The vector space of all triple analytic sequences are usually denoted by Λ^3.
A sequence $x = (x_{mnk})$ is called triple entire sequence if

$x = (x_{mnk})$. as $m,n,k \to \infty$.

The vector space of all triple entire sequences are usually denoted by Γ^3. The space Λ^3 and Γ^3 is a metric space with the metric

$$d(x,y) = sup_{m,n,k} \left\{ |x_{mnk} - y_{mnk}|^{\frac{1}{m+n+k}} : m,n,k : 1,2,3,... \right\}, \tag{1}$$

For all $x = \{x_{mnk}\}$ and $y = \{y_{mnk}\}$ in Γ^3. Let $x = (x_{mnk})$.

Consider a triple sequence $x = (x_{mnk})$. The $(m,n,k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by $x^{[m,n,k]} = \sum_{i,j,q=0}^{m,n,k} x_{ijq} \delta_{ijq}$ for all $m,n,k \in N$,

$$\delta_{mnk} = \begin{pmatrix} 0 & 0 & ...0 & 0 & ... \\ 0 & 0 & ...0 & 0 & ... \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & ...1 & 0 & ... \\ 0 & 0 & ...0 & 0 & ... \end{pmatrix}$$

with 1 in the $(m,n,k)^{th}$ position and zero otherwise.

Let M and Φ are mutually complementary Orlicz functions. Then, we have:

(i) For all $u, y \geq 0,$

$$uy \leq M(u) + \Phi(y), \quad (Young's\ inequality) \tag{2}$$

(ii) For all $u \geq 0,$

$$u\eta(u) = M(u) + \Phi(\eta(u)). \tag{3}$$

(iii) For all $u \geq 0,$ and $0 < \lambda < 1,$

$$M(\lambda u) \leq \lambda M(u). \tag{4}$$

Lindenstrauss and Tzafriri [26,27] used the idea of Orlicz function to construct Orlicz sequence space

$$\ell_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \text{ for some } \rho > 0 \right\},$$

The space ℓ_M with the norm

$$\|x\| = inf\left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \le 1 \right\},$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p \, (1 \le p < \infty),$ the spaces ℓ_M coincide with the classical sequence space ℓ_p.

A sequence $f = (f_{mnk})$ of Orlicz function is called a Musielak -Orlicz function. A sequence $g = (g_{mnk})$ defined by

$g_{mnk}(v) = sup\{|v|u - (f_{mnk})(u) : u \ge 0\},\ m,n,k = 1,2,\cdots$

is called the complementary function of a Musielak-Orlicz function f. For a given MusielakOrlicz function f, the Musielak-Orlicz sequence space t_f. [see 20]

$t_f = \left\{ x \in w^3 : M_f\left(|x_{mnk}|\right)^{1/m+n+k} \to 0 \, as \, m,n,k \to \infty \right\},$

where M_f is a convex modular defined by

$M_f(x) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} f_{mnk}\left(|x_{mnk}|\right)^{1/m+n+k}, x = (x_{mnk}) \in t_f.$

We consider t_f equipped with the Luxemburg metric

$$d(x,y) = sup_{m,n,k}\left\{ inf\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} f_{mnk}\left(\frac{|x_{mnk}|^{1/m+n+k}}{mnk}\right)\right) \le 1 \right\}.$$

2. Definition and preliminaries

A sequence $x = (x_{mnk})$ is said to be triple analytic if $sup_{m,n,k}|x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$ The vector space of all triple analytic sequences is usually denoted by Λ^3. A sequence $x = (x_{mnk})$ is called triple entire sequence if $|x_{mnk}|^{\frac{1}{m+n+k}} \to 0$ as $m,n,k \to \infty.$ The vector space of triple entire sequences is usually denoted by Γ^3.

Let w^3 denote the set of all complex double sequences $x=\left(x_{mnk}\right)_{m,n,k=1}^{\infty}$ and $M:\left[0,\infty\right)\to\left[0,\infty\right)$ be an Orlicz function. Given a triplesequence, $x\in w^3$. Definethesets

$$\Gamma_M^3=\left\{x\in w^3:\left(M\left(\frac{\left|x_{mnk}\right|^{\frac{1}{m+n+k}}}{\rho}\right)\right)\to 0\, as\, m,n,k\to\infty\ for\ some\ \rho>0\right\} \text{ and}$$

$$\Lambda_M^3=\left\{x\in w^3:sup_{m,n,k\geq 1}\left(M\left(\frac{\left|x_{mnk}\right|^{\frac{1}{m+n+k}}}{\rho}\right)\right)<\infty\ for\ some\ \rho>0\right\}.$$

The space Λ_M^3 is a metric space with the metric

$$d\left(x,y\right)=inf\left\{\rho>0:sup_{m,n,k\geq 1}\left(M\left(\frac{\left|x_{mnk}-y_{mnk}\right|}{\rho}\right)\right)^{\frac{1}{m+n+k}}\leq 1\right\}$$

The space Γ_M^3 is a metric space with the metric

$$\tilde{d}\left(x,y\right)=inf\left\{\rho>0:sup_{m,n,k\geq 1}\left(M\left(\frac{\left|x_{mnk}-y_{mnk}\right|}{\rho}\right)\right)^{\frac{1}{m+n+k}}\leq 1\right\}$$

Let $n\in N$ and X be a real vector space of dimension w, where $n\leq m.$A real valued function $d_p(x_1,\ldots,x_n)=\|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_p$ on X satisfying the following four conditions:

(i) $\|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_p=0$ if and and only if $d_1(x_1,0),\ldots,d_n(x_n,0)$ are linearly dependent,

(ii) $\|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_p$ is invariant under permutation,

(iii) $\|(\alpha d_1(x_1,0),\ldots,d_n(x_n,0))\|_p=\left|\alpha\right|\|(d_1(x_1,0),\ldots,d_n(x_n,0))\|_p,\alpha\in R$

(iv) $d_p\left((x_1,y_1),(x_2,y_2)\cdots(x_n,y_n)\right)=\left(d_X(x_1,x_2,\cdots x_n)^p+d_Y(y_1,y_2,\cdots y_n)^p\right)^{1/p}$ $for 1\leq p<\infty;$ (or)

(v) $d\left((x_1,y_1),(x_2,y_2),\cdots(x_n,y_n)\right):=\sup\left\{d_X(x_1,x_2,\cdots x_n),d_Y(y_1,y_2,\cdots y_n)\right\}$,

for $x_1,x_2,\cdots x_n\in X,y_1,y_2,\cdots y_n\in Y$ is called the p product metric of the Cartesian product of n metric spaces is the p norm of the n-vector of the norms of the n subspaces.

A trivial example of p product metric of n metric space is the p norm space is $X = R$ equipped with the following Euclidean metric in the product space is the p norm:

$$\| (d_1(x_1,0),\ldots,d_n(x_n,0)) \|_E = sup\left(| \, det\,(d_{mn}\left(x_{mn},0\right)) \, |\right) =$$

$$= sup\left(\begin{vmatrix} d_{11}\left(x_{11},0\right) & d_{12}\left(x_{12},0\right) & \ldots & d_{1n}\left(x_{1n},0\right) \\ d_{21}\left(x_{21},0\right) & d_{22}\left(x_{22},0\right) & \ldots & d_{2n}\left(x_{1n},0\right) \\ . & & & \\ . & & & \\ . & & & \\ d_{n1}\left(x_{n1},0\right) & d_{n2}\left(x_{n2},0\right) & \ldots & d_{nn}\left(x_{nn},0\right) \end{vmatrix}\right)$$

where $x_i = \left(x_{i1},\cdots x_{in}\right) \in R^n$ for each $i = 1,2,\cdots n.$

If every Cauchy sequence in X converges to some $L \in X$, then X is said to be complete with respect to the p-metric. Any complete p-metric space is said to be p-Banach metric space.

2.1. Definition

Let X be a linear metric space. A function $\rho : X \to R$ is called paranorm, if

(1) $\rho\left(x\right) \geq 0,$ for all $x \in X$;

(2) $\rho\left(-x\right) = \rho\left(x\right),$ for all $x \in X$;

(3) $\rho\left(x+y+z\right) \leq \rho\left(x\right)+\rho\left(y\right)+\rho\left(z\right),$ for all $x,y,z \in X$;

(4) If $\left(\sigma_{mnk}\right)$ is a sequence of scalars with $\sigma_{mnk} \to \sigma$ as $m,n,k \to \infty$ and $\left(x_{mnk}\right)$ is a sequence of vectors with $\rho\left(x_{mnk}-x\right) \to 0$ as $m,n,k \to \infty,$ then $\rho\left(\sigma_{mnk}x_{mnk}-\sigma x\right) \to 0$ as $m,n,k \to \infty.$

2.2. Definition

The triple sequence $\theta_{i,\ell,j} = \left\{\left(m_i,n_\ell,k_j\right)\right\}$ is called triple lacunary if there exist three increasing sequences of integers such that

$m_0 = 0, h_i = m_i - m_{r-1} \to \infty$ as $i \to \infty$ and

$n_0 = 0, \overline{h_\ell} = n_\ell - n_{\ell-1} \to \infty$ as $\ell \to \infty.$

$k_0 = 0, \overline{h_j} = k_j - k_{j-1} \to \infty$ as $j \to \infty.$

Let $m_{i,\ell,j} = m_i n_\ell k_j, h_{i,\ell,j} = h_i \overline{h_\ell}\,\overline{h_j},$ and $\theta_{i,\ell,j}$ is determine by

$$I_{i,\ell,j}=\left\{(m,n,k):m_{i-1}<m<m_i \text{ and } n_{\ell-1}<n\le n_\ell \text{ and } k_{j-1}<k\le k_j\right\},$$

$$q_k=\frac{m_k}{m_{k-1}},\ \overline{q_\ell}=\frac{n_\ell}{n_{\ell-1}},\ \overline{q_j}=\frac{k_j}{k_{j-1}}.$$

The notion of $\lambda-$triple entire and triple analytic sequences as follows: Let $\lambda=(\lambda_{mnk})_{m,n,k=0}^{\infty}$ be a strictly increasing sequences of positive real numbers tending to infinity, that is

$$0<\lambda_{000}<\lambda_{111}<\cdots \text{ and } \lambda_{mnk}\to\infty \text{ as } m,n,k\to\infty$$

and said that a sequence $x=(x_{mnk})\in w^3$ is $\lambda-$convergent to 0, called a the $\lambda-$limit of x, if $\mu_{mnk}(x)\to 0 \text{ as } m,n,k\to\infty,$ where

$$\mu_{mnk}(x)=\frac{1}{\varphi_{rs}}\sum_{m\in I_{rst}}\sum_{n\in I_{rst}}\sum_{k\in I_{rst}}\left(\Delta^{m-1}\lambda_{m,n}\right)-\left(\Delta^{m-1}\lambda_{m,n+1}\right)-\left(\Delta^{m-1}\lambda_{m,n+2}\right)-$$
$$\left(\Delta^{m-1}\lambda_{m+1,n}\right)-\left(\Delta^{m-1}\lambda_{m+1,n+1}\right)-\left(\Delta^{m-1}\lambda_{m+1,n+2}\right)-\left(\Delta^{m-1}\lambda_{m+2,n}\right)-$$
$$-\left(\Delta^{m-1}\lambda_{m+2,n+1}\right)-\left(\Delta^{m-1}\lambda_{m+2,n+2}\right)\left|x_{mn}\right|^{1/m+n+k}.$$

The sequence $x=(x_{mnk})\in w^3$ is $\lambda-$triple analytic if $sup_{uvs}\left|\mu_{mnk}(x)\right|<\infty.$ If $lim_{mnk}x_{mnk}=0$ in the ordinary sense of convergence, then

$$lim_{mnk}\frac{1}{\varphi_{rs}}\sum_{m\in I_{rst}}\sum_{n\in I_{rst}}\sum_{k\in I_{rst}}\left(\Delta^{m-1}\lambda_{m,n}\right)-\left(\Delta^{m-1}\lambda_{m,n+1}\right)-\left(\Delta^{m-1}\lambda_{m,n+2}\right)-$$
$$-\left(\Delta^{m-1}\lambda_{m+1,n}\right)-\left(\Delta^{m-1}\lambda_{m+1,n+1}\right)-\left(\Delta^{m-1}\lambda_{m+1,n+2}\right)-\left(\Delta^{m-1}\lambda_{m+2,n}\right)-$$
$$-\left(\Delta^{m-1}\lambda_{m+2,n+1}\right)-\left(\Delta^{m-1}\lambda_{m+2,n+2}\right)\left|x_{mn}\right|^{1/m+n+k}=0$$

This implies that

$$lim_{mnk}\left|\mu_{mnk}(x)-0\right|=lim_{mnk}\frac{1}{\varphi_{rs}}\sum_{m\in I_{rst}}\sum_{n\in I_{rst}}\sum_{k\in I_{rst}}\left(\Delta^{m-1}\lambda_{m,n}\right)-$$
$$-\left(\Delta^{m-1}\lambda_{m,n+1}\right)-\left(\Delta^{m-1}\lambda_{m,n+2}\right)-\left(\Delta^{m-1}\lambda_{m+1,n}\right)-$$
$$\left(\Delta^{m-1}\lambda_{m+1,n+1}\right)-\left(\Delta^{m-1}\lambda_{m+1,n+2}\right)-\left(\Delta^{m-1}\lambda_{m+2,n}\right)-$$
$$-\left(\Delta^{m-1}\lambda_{m+2,n+1}\right)-\left(\Delta^{m-1}\lambda_{m+2,n+2}\right)\left|x_{mn}-0\right|^{1/m+n+k}=0$$

which yields that $lim_{uvs}\mu_{mnk}(x)=0$ and hence $x=(x_{mnk})\in w^3$ is $\lambda-$convergent to 0.

Let $I^3 -$be an admissible ideal of $3^{N\times N\times N}, \theta_{rst}$ be a triple lacunary sequence, $f = (f_{mnk})$ be a Musielak-Orlicz function and $\left(X, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)$ be a p-metric space, $q = (q_{mnk})$ be triple analytic sequence of strictly positive real numbers. By $w^3(p-X)$ we denote the space of all sequences defined over $\left(X, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)$.

The following inequality will be used throughout the paper. If $0 \le q_{mnk} \le sup\, q_{mnk} = H, K = max\left(1, 2^{H-1}\right)$ then

$$|a_{mnk} + b_{mnk}|^{q_{mnk}} \le K\left\{|a_{mnk}|^{q_{mnk}} + |b_{mnk}|^{q_{mnk}}\right\} \tag{5}$$

for all m, n, k and $a_{mnk}, b_{mnk} \in C.$ Also $|a|^{q_{mnk}} \le max\left(1, |a|^H\right)$ for all $a \in C.$

In the present paper, we define the following sequence spaces:

$$\left[\Gamma^{3q}_{f\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_p\right]^{I^3}_{\theta_{rst}} =$$

$$\left\{r,s,t \in I_{rst} : \left[f_{mnk}\left(\left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}} \ge \mathrm{T}\right\} \in I^3$$

$$\left[\Lambda^{3q}_{f\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_p\right]^{I^3}_{\theta_{rst}}$$

$$= \left\{r,s,t \in I_{rst} : \left[f_{mnk}\left(\left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}} \ge K\right\} \in I^3,$$

If we take $f_{mnk}(x) = x,$ we get

$$\left[\Gamma^{3q}_{f\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_p\right]^{I^3}_{\theta_{rst}} =$$

$$\left\{r,s,t \in I_{rst} : \left[\left(\left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}} \ge \mathrm{T}\right\} \in I^3,$$

$$\left[\Lambda^{3q}_{f\mu}, \left\|\left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_p\right]^{I^3}_{\theta_{rst}} =$$

$$\left\{r,s,t \in I_{rst} : \left[\left(\left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}} \ge K\right\} \in I^3,$$

If we take $q = (q_{mnk}) = 1,$ we get

$$\left[\Gamma^3_{f\mu}, \left\|\left(d\left(x_1,0\right), d\left(x_2,0\right), \cdots, d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_{p}\right]^{I}_{\theta_{rst}} =$$

$$\left\{r,s,t \in I_{rst} : \left[f_{mnk}\left(\left\|\mu_{mnk}\left(x\right), \left(d\left(x_1,0\right), d\left(x_2,0\right), \cdots, d\left(x_{n-1},0\right)\right)\right\|_{p}\right)\right] \geq \text{т}\right\} \in I^3,$$

$$\left[\Lambda^3_{f\mu}, \left\|\left(d\left(x_1,0\right), d\left(x_2,0\right), \cdots, d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}} =$$

$$\left\{r,s,t \in I_{rst} : \left[f_{mnk}\left(\left\|\mu_{mnk}\left(x\right), \left(d\left(x_1,0\right), d\left(x_2,0\right), \cdots, d\left(x_{n-1},0\right)\right)\right\|_{p}\right)\right] \geq K\right\} \in I^3,$$

In the present paper we plan to study some topological properties and inclusion relation between the above defined sequence spaces.

$$\left[\Gamma^{3q}_{f\mu}, \left\|\left(d\left(x_1,0\right), d\left(x_2,0\right), \cdots, d\left(x_{n-1}\right)\right)\right\|^{\varphi}_{p}\right]^{I^2}_{\theta_{rs}}$$

and

$$\left[\Lambda^{3q}_{f\mu}, \left\|\left(d\left(x_1,0\right), d\left(x_2,0\right), \cdots, d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}$$

which we shall discuss in this paper.

3. Main results

Theorem 3.1. Let $f = \left(f_{mnk}\right)$ be a Musielak-Orlicz function, $q = \left(q_{mnk}\right)$ be a triple analytic sequence of strictly positive real numbers, the sequence spaces

$$\left[\Gamma^{3q}_{f\mu}, \left\|\left(d\left(x_1,0\right), d\left(x_2,0\right), \cdots, d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}$$

and

$$\left[\Lambda^{3q}_{f\mu}, \left\|\left(d\left(x_1,0\right), d\left(x_2,0\right), \cdots, d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}$$

are linear spaces.

Proof: It is routine verification. Therefore the proof is omitted.

Theorem 3.2. Let $f = \left(f_{mnk}\right)$ be a Musielak-Orlicz function, $q = \left(q_{mnk}\right)$ be a triple analytic sequence of strictly positive real numbers, the sequence space

$\left[\Gamma^{3q}_{f\mu}, \left\|\left(d\left(x_1,0\right), d\left(x_2,0\right), \cdots, d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}$ is a paranormed space with respect to the paranorm defined by

$$g(x) = inf$$
$$\left\{\left[f_{mnk}\left(\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}} \leq 1\right\}.$$

Proof: Clearly $g(x) \geq 0$ for

$$x = (x_{mnk}) \in \left[\Gamma^{3q}_{f\mu}, \left\|\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|^{\varphi}_p\right]^{I^3}_{\theta_{rst}}.$$

Since $f_{mnk}(0) = 0,$ we get $g(0) = 0.$

Conversely, supposethat $g(x) = 0,$ then

$$inf\left\{\left[f_{mnk}\left(\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}} \leq 1\right\} = 0.$$

Suppose that $\mu_{mnk}(x) \neq 0$ for each $m,n,k \in N.$ Then

$$\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|^{\varphi}_p \to \infty.$$

It follows that

$$\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\right)^{1/H} \to \infty$$

which is a contradiction. Therefore $\mu_{mnk}(x) = 0.$ Let

$$\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\right)^{1/H} \leq 1$$

and

$$\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(y),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\right)^{1/H} \leq 1$$

Then by using Minkowski's inequality, we have

$$\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(x+y),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\right)^{1/H} \leq$$

$$\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(x),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\right)^{1/H} +$$

$$\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(y),\left(d(x_1,0),d(x_2,0),\cdots,d(x_{n-1},0)\right)\right\|_p\right)\right]^{q_{mnk}}\right)^{1/H}.$$

Sowehave

$$g\left(x+y\right)=$$
$$=inf\left\{\left[f_{mnk}\left(\left\|\mu_{mnk}\left(x+y\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|_p\right)\right]^{q_{mnk}}\leq 1\right\}\leq$$
$$inf\left\{\left[f_{mnk}\left(\left\|\mu_{mnk}\left(x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|_p\right)\right]^{q_{mnk}}\leq 1\right\}+$$
$$inf\left\{\left[f_{mnk}\left(\left\|\mu_{mnk}\left(y\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|_p\right)\right]^{q_{mnk}}\leq 1\right\}.$$

Therefore,

$$g\left(x+y\right)\leq g\left(x\right)+g\left(y\right).$$

Finally, to prove that the scalar multiplication is continuous. Let λ be any complex number. By definition,

$$g\left(\lambda x\right)=inf\left\{\left[f_{mnk}\left(\left\|\mu_{mnk}\left(\lambda x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|_p\right)\right]^{q_{mnk}}\leq 1\right\}.$$

Then

$$g\left(\lambda x\right)=$$
$$=inf\left\{((\left|\lambda\right|t)^{q_{mnk}/H}:\left[f_{mnk}\left(\left\|\mu_{mnk}\left(\lambda x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|_p\right)\right]^{q_{mnk}}\leq 1\right\}$$

where $t=\frac{1}{\left|\lambda\right|}$. Since $\left|\lambda\right|^{q_{mnk}}\leq max\left(1,\left|\lambda\right|^{supq_{mnk}}\right)$, wehave

$$g\left(\lambda x\right)\leq max\left(1,\left|\lambda\right|^{supq_{mnk}}\right)$$
$$inf\left\{t^{q_{mnk}/H}:\left[f_{mnk}\left(\left\|\mu_{mnk}\left(\lambda x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|_p\right)\right]^{q_{mnk}}\leq 1\right\}$$

This completes the proof.

Theorem 3.3.

(i) If the MusielakOrlicz function $\left(f_{mnk}\right)$ satisfies Δ_2-condition, then

$$\left[\Gamma^{3q}_{f\mu},\left\|\mu_{mnk}\left(x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_p\right]^{I^{3\alpha}}_{\theta_{rst}}=$$
$$\left[\Gamma^{3q\mu}_{g},\left\|\mu_{uvs}\left(x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_p\right]^{I^{3}}_{\theta_{rst}}.$$

(ii) If the MusielakOrlicz function (g_{mnk}) satisfies $\Delta_2 -$condition, then $\left[\Gamma^{3q\mu}_{g}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^{3\alpha}}_{\theta_{rst}} =$

$\left[\Gamma^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^{3}}_{\theta_{rst}}$

Proof: Let the MusielakOrlicz function (f_{mnk}) satisfies $\Delta_2 -$condition, we get

$$\left[\Gamma^{3q\mu}_{g}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^{3}}_{\theta_{rst}} \subset \left[\Gamma^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^{3\alpha}}_{\theta_{rst}} \quad (6)$$

To prove the inclusion

$\left[\Gamma^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^{3\alpha}}_{\theta_{rst}} \subset$

$\left[\Gamma^{3q\mu}_{g}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^{3}}_{\theta_{rst}}$,

Let $a \in \left[\Gamma^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^{3\alpha}}_{\theta rst}$.

Then for all $\{x_{mnk}\}$ with

$(x_{mnk}) \in \left[\Gamma^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^{3}}_{\theta_{rst}}$

we have

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} |x_{mnk} a_{mnk}| < \infty. \quad (7)$$

Since the MusielakOrlicz function (f_{mnk}) satisfies $\Delta_2 -$condition, then

$(y_{mnk}) \in \left[\Gamma^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^{3}}_{\theta_{rst}}$,

we get

$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} \left| \frac{\varphi_{rst} y_{mnk} a_{mnk}}{\Delta^m \lambda_{mnk}} \right| < \infty.$ by (3.2). Thus

$$\left(\varphi_{rst} a_{mnk}\right) \in \left[\Gamma^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}} =$$

$$\left[\Gamma^{3q\mu}_{g}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}$$

and hence

$$\left(a_{mnk}\right) \in \left[\Gamma^{3q\mu}_{g}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}.$$

This gives that

$$\begin{aligned}&\left[\Gamma^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^{3\alpha}}_{\theta_{rst}} \subset \\ &\subset \left[\Gamma^{3q\mu}_{g}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rs}}\end{aligned} \tag{8}$$

we are granted with (6) and (8)

$$\left[\Gamma^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^{3\alpha}}_{\theta_{rst}} =$$

$$\left[\Gamma^{3q\mu}_{g}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}.$$

(ii) Similarly, one can prove that

$$\left[\Gamma^{3q\mu}_{g}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^{3\alpha}}_{\theta_{rst}} \subset$$

$$\left[\Gamma^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}$$

if the MusielakOrlicz function
$\left(g_{mnk}\right)$ satisfies Δ_2-condition.

Proposition 3.4. If $0 < q_{mnk} < p_{mnk} < \infty$ for each m, n and k then,

$$\left[\Lambda^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}} \subseteq$$

$$\left[\Lambda^{3p}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}.$$

Proof: The proof is standard, so we omit it.

Proposition 3.5.

(i) If $0 < infq_{mnk} \leq q_{mnk} < 1$ then

$$\left[\Lambda^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}} \subset$$

$$\left[\Lambda^{3}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}.$$

(ii) If $1 \leq q_{mnk} \leq supq_{mnk} < \infty,$ then

$$\left[\Lambda^{3}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}} \subset$$

$$\subset \left[\Lambda^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}.$$

Proof: The proof is standard, so we omit it.

Proposition 3.6. Let $f' = \left(f'_{mnk}\right)$ and $f'' = \left(f''_{mnk}\right)$ are sequences of Musielak functions, we have

$$\left[\Lambda^{3q}_{f'\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}} \bigcap$$

$$\left[\Lambda^{3q}_{f''\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}} \subseteq$$

$$\left[\Lambda^{3q}_{f'+f''\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}.$$

Proof: The proof is easy, so we omit it.

Proposition 3.7. For any sequence of Musielak functions $f = \left(f_{mnk}\right)$ $q = \left(q_{mnk}\right)$ be triple analytic sequence of strictly positive real numbers. Then

$$\left[\Gamma^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}} \subset$$

$$\left[\Lambda^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}.$$

Proof: The proof is easy, so we omit it.

Proposition 3.8.

The sequence space $\left[\Lambda^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}$ is solid.

Proof: Let

$$x=\left(x_{mnk}\right)\in\left[\Lambda^{3q}_{f\mu},\left\|\mu_{mnk}\left(x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}},\text{ (i.e.)}$$

$$sup_{mnk}\left[\Lambda^{3q}_{f\mu},\left\|\mu_{mnk}\left(x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}<\infty.$$

Let $\left(\alpha_{mnk}\right)$ be triple sequence of scalars such that $\left|\alpha_{mnk}\right|\le 1$ for all $m,n,k\in N\times N\times N.$ Then we get

$$sup_{mnk}\left[\Lambda^{3q}_{f\mu},\left\|\mu_{mnk}\left(\alpha x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}\le$$

$$\sup_{mnk}\left[\Lambda^{3q}_{f\mu},\left\|\mu_{mnk}\left(x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}.$$ This completes the proof.

Proposition 3.9. The sequence space

$$\left[\Lambda^{3q}_{f\mu},\left\|\mu_{mnk}\left(x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}$$ is monotone.

Proof: The proof follows from Proposition 3.8.

Proposition 3.10. If $f=\left(f_{mnk}\right)$ be any Musielak function. Then

$$\left[\Lambda^{3q}_{f\mu},\left\|\mu_{mnk}\left(x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|^{\varphi^*}_{p}\right]^{I^3}_{\theta_{rst}}\subset$$

$$\subset\left[\Lambda^{3q}_{f\mu},\left\|\mu_{mnk}\left(x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|^{\varphi^{**}}_{p}\right]^{I^3}_{\theta_{rst}}$$

if and only if $sup_{r,s,t\ge 1}\frac{\varphi^{*}_{rst}}{\varphi^{**}_{rst}}<\infty.$

Proof: Let $x\in\left[\Lambda^{3q}_{f\mu},\left\|\mu_{mnk}\left(x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|^{\varphi^*}_{p}\right]^{I^3}_{\theta_{rst}}$ and

$N=sup_{r,s,t\ge 1}\frac{\varphi^{*}_{rst}}{\varphi^{**}_{rst}}<\infty.$ Then we get

$$\left[\Lambda^{3q}_{f\mu},\left\|\mu_{mnk}\left(x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|^{\varphi^{**}_{rst}}_{p}\right]^{I^3}_{\theta_{rst}}=\mathrm{t}$$

$$N\left[\Lambda^{3q}_{f\mu},\left\|\mu_{mnk}\left(x\right),\left(d\left(x_1,0\right),d\left(x_2,0\right),\cdots,d\left(x_{n-1},0\right)\right)\right\|^{\varphi^{*}_{rst}}_{p}\right]^{I^3}_{\theta_{rst}}=0.$$

Thus, $x \in \left[\Lambda^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0) \right) \right\|^{\varphi^{**}}_p \right]^{I^3}_{\theta_{rst}}$.

Conversely, suppose that

$$\left[\Lambda^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0) \right) \right\|^{\varphi^{*}}_p \right]^{I}_{N_\theta} \subset$$

$$\left[\Lambda^{3qu}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0) \right) \right\|^{\varphi^{**}}_p \right]^{I^3}_{\theta_{rst}}$$ and

$$x \in \left[\Lambda^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0) \right) \right\|^{\varphi^{*}}_p \right]^{I^3}_{\theta_{rst}}.$$ Then

$$\left[\Lambda^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0) \right) \right\|^{\varphi^{*}}_p \right]^{I^3}_{\theta_{rst}} < \tau,$$ for every $\tau > 0$.

Suppose that $sup_{r,s,t\geq 1} \frac{\varphi^{*}_{rst}}{\varphi^{**}_{rst}} = \infty$, then there exists a sequence of members (rst_{abc}) such that $lim_{a,b,c\to\infty} \frac{\varphi^{*}_{abc}}{\varphi^{**}_{abc}} = \infty$. Hence, we have

$$\left[\Lambda^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0) \right) \right\|^{\varphi^{*}_{rst}}_p \right]^{I^3}_{\theta_{rst}} = \infty.$$ Therefore

$$x \notin \left[\Lambda^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0) \right) \right\|^{\varphi^{**}}_p \right]^{I^3}_{\theta_{rst}},$$ which is a contradiction. This completes the proof.

Proposition 3.11. If $f = (f_{mnk})$ be any Musielak function. Then

$$\left[\Lambda^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0) \right) \right\|^{\varphi^{*}}_p \right]^{I^3}_{\theta_{rst}} =$$

$$\left[\Lambda^{3q}_{f\mu}, \left\| \mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0) \right) \right\|^{\varphi^{**}}_p \right]^{I^3}_{\theta_{rst}}$$

if and only if $sup_{r,s,t\geq 1} \frac{\varphi^{*}_{rst}}{\varphi^{**}_{rst}} < \infty, sup_{r,s,t\geq 1} \frac{\varphi^{**}_{rst}}{\varphi^{*}_{rst}} > \infty$.

Proof: It is easy to prove so we omit.

Proposition 3.12. The sequence space

$$\left[\Gamma^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}$$ is not solid.

Proof: The result follows from the following example.

Example: Consider

$$x = (x_{mnk}) = \begin{pmatrix} 1 & 1 & \ldots & 1 \\ 1 & 1 & \ldots & 1 \\ . & & & \\ . & & & \\ . & & & \\ 1 & 1 & \ldots & 1 \end{pmatrix} \in$$

$$\in \left[\Gamma^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}.$$

Let $\alpha_{mnk} = \begin{pmatrix} -1^{m+n+k} & -1^{m+n+k} & \ldots & -1^{m+n+k} \\ -1^{m+n+k} & -1^{m+n+k} & \ldots & -1^{m+n+k} \\ . & & & \\ . & & & \\ . & & & \\ -1^{m+n+k} & -1^{m+n+k} & \ldots & -1^{m+n+k} \end{pmatrix}$, for all $m,n,k \in N$.

Then $\alpha_{mnk} x_{mnk} \notin \left[\Gamma^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}$.

Hence, $\left[\Gamma^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}$ is not solid.

Proposition 3.13.

The sequence space $\left[\Gamma^{3q}_{f\mu}, \left\|\mu_{mnk}(x), \left(d(x_1,0), d(x_2,0), \cdots, d(x_{n-1},0)\right)\right\|^{\varphi}_{p}\right]^{I^3}_{\theta_{rst}}$ is not monotone.

Proof: The proof follows from Proposition 3.12.

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Orliç funksiyasiilə təyin olunan p - metric fəzalarda üçqat lakunar statistic yiğilma

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XÜLASƏ

Bu işdə biz üçqat lakunar statistik yığılma və Γ^3 -də Orliç funksiyaları ilə təyin edilən təsadüfi p -metrik fəzalarda üçqat lakunar Koşi ardıcıllıqları anlayışlarını daxil edir və onları öyrənirik. İşdə məlum nəticələri ümumiləşdirən teoremlər isbat edilmişdir.

Açar sözlər: analitik ardıcıllıq, üçqat ardıcıllıq, Musieleka-Orliç funksiyası, təsadüfi p -metrik fəza,lakunar ardıcıllıqlar, statistik yığılma.

Тройная лакунарная статистическая сходимость по Γ^3 на p - метрических пространтвах определеяемых функцией Орлица

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Резюме

В данной работе даем определение и изучаем понятие тройной лакунарной статистической сходимости и тройную лакунарную статистичесую последовательность Коши, в случайном в Γ^3 на p -метрике пространств, определяемых функцией Орлича и доказываем некоторые теоремы, которые обобщают известные результаты.

Ключевые слова: аналитическая последовательность, тройные последовательности, пространство, функция Мусиэлака - Орлича, случайные p - метрические пространства, лакунарная последовательность, статистическая сходимость.