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REMARK ON THE PAPER ENTITLED "A NEGATIVE EXPONENTIAL SOLUTION FOR THE MATRIX RICCATI EQUATION"

N.S. Hajiyeva [1]

[1] Institute of Applied Mathematics, Baku State University, Baku , Azerbaijan
e-mail: n_hajiyeva@yahoo.com

Abstract. A new formula is presented for solving algebraic matrix Riccati equations using the eigenvectors of the corresponding Hamiltonian matrices. Unlike the approach in known work, the proposed method enables the solution of families of linear–quadratic optimal control problems over an infinite time horizon. Numerical examples are provided to demonstrate the validity and effectiveness of the obtained results.

Keywords: Algebraic matrix Riccati equations, Hamiltonian matrix, eigenvector, Euler-Lagrange equations.

AMS Subject Classification: 15A24, 34C25, 65F30, 93D15.

1. Introduction

Adopting the notation

$$H = \begin{bmatrix} -F & GC^{-1}G' \\ R & F' \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \begin{bmatrix} \Lambda & 0 \\ 0 & -\Lambda \end{bmatrix} \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}^{-1}$$

for solving the algebraic matrix Riccati equation (AMRE) [1-5]

$$SF + F'S - SGC^{-1}G'S + R = 0, \tag{1}$$

a new formula is derived

$$S_N = (V_{11}V_{21}^{-1}H_{22} - H_{12})^{-1}(H_{11} - V_{11}V_{21}^{-1}H_{21}), \tag{2}$$

where the known constant matrices $F, G, C = C' > 0, R = R' \geq 0$ have corresponding dimensions, the pair (F, G) is stabilizable, and the pair $(R^{\frac{1}{2}}, F')$ is detectable [6-10]. The prime symbol $(')$ denotes matrix transposition. The matrix Λ containing the eigenvalues of Hamiltonian matrix H lie on the right half plane, the matrix $\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$ corresponding eigenvectors and

$$(-H)^{-1} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}.$$

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Formula (2) successfully solves a series of optimal regulation problems. However the formula [18]

$$S_V = V_{21}V_{11}^{-1} \tag{3}$$

for solving the matrix algebraic Riccati equations (1) does not give a positive result when V_{11}^{-1} does not exist, i.e. it does not solve the equation (1).
Formula (3) shows that when V_{11}^{-1} does not exist or $\det V_{11}$ is close to zero, the relation (2) works more correctly and vice versa when from (2) $\det V_{21}$ is close to zero or V_{21}^{-1} does not exist, it is better to work with formula (3).
Now we illustrate this with the following examples for the problem of optimizing weak control of a weakly damped system [1, 11-16].
Note that in future the problem in [18] will be solved using Schur method [17], the method of orthogonalization 3].

Example 1. Let

$$F = \begin{bmatrix} 1.5\cdot 10^{-3} & -10^{-3} \\ 5\cdot 10^{-4} & 0 \end{bmatrix}, GC^{-1}G' = \begin{bmatrix} 5.15\cdot 10^{-15} & -7.6\cdot 10^{-15} \\ -0.1162\cdot 10^{-15} & 1.25\cdot 10^{-14} \end{bmatrix},$$
$$R = \begin{bmatrix} 36 & 20 \\ 20 & 12 \end{bmatrix}.$$

Note that, the norm of residual of equation (1) corresponding solution (2) will be as

$$R_1 = 2.1324\cdot 10^{-6}.$$

Analogously, the norm of residual of equation (1) corresponding solution (3) will be as

$$R_2 = 39.53.$$

Example 2. Let

$$F = \begin{bmatrix} -3\cdot 10^{-5} & 7\cdot 10^{-4} \\ -4\cdot 10^{-4} & 8\cdot 10^{-4} \end{bmatrix}, GC^{-1}G' = \begin{bmatrix} 6.4761\cdot 10^{-20} & -1.1619\cdot 10^{-19} \\ -1.1619\cdot 10^{-19} & 2.1904\cdot 10^{-19} \end{bmatrix},$$
$$R = \begin{bmatrix} 24 & 16 \\ 16 & 12 \end{bmatrix}.$$

Note that, the norm of residual of equation (1) corresponding solution (2) will be as $R_1 = 1.0868\cdot 10^{-4}$.
Analogously, the norm of residual of equation (1) corresponding solution (3) will be as

$$R_2 = 2.7321\cdot 10^{5}.$$

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